Inventory pricing strategy considering product returns for deteriorating items with multi-variate dependent demand

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Abstract: This study discusses an optimal inventory model for rapidly decaying products considering product returns. In this study, the demand is influenced by selling price, timing, and advertisements per planning period. Product returns is regarded as a function of the product's demand and price. The shortages are acceptable and partially backlogged in the proposed model. This inventory model aims to develop a profit function to obtain the optimal selling price of the product and replenishment schedule with maximum profit. Moreover, a novel algorithm is proposed to determine the optimal results. An optimal replenishment schedule exists for a given selling price. Finally, the optimality of the function is proved mathematically and graphically to substantiate the appropriateness of the solution algorithm and method. In addition, a numerical illustration is used to elaborate the solution procedure, followed by a meticulous analysis of various parameters.

Keywords: inventory optimisation; deterioration; shortages; product returns; dependent demand.

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1 Introduction

Inventory management is an important field of logistics and production management, and it has a significant impact on the profits of any company. In inventory management, customer returns have become a significant concern for manufacturers and retailers in recent years. Competitive market places are characterised by customer returns from customers to retailers. Items returned to the manufacturer or retailer after the selling season is referred to as customer returns. Product returns occurs due to the customer's shifting attitude toward the product or things with problems or an exchange for a different item. Liquidation and closeout lots sometimes include customer return products. Hess and Mayhew (1997) used regression methods to describe the returns for a larger direct market and examined the concerns of customer returns. Pasternack (2008) examined the newsvendor issue for a seasonal product where sellers were required to return a particular proportion of the lot size to the producer. El Saadany and Jaber (2010) created a production/re-manufacturing inventory model with a price and quality-dependent return rate. Mesak et al. (2011) presented an optimal advertising approach for subscriber service cost learning and consumer's rejection over time. This study considers product returns with product deterioration to construct the inventory model.

1.1 Motivation

Inventory management is critical in today's competitive market to cut costs, enhance customer service, and create a balance between costs and services, giving a company a competitive edge. A novel part of the research has been dedicated to area inventory management over the past few decades. Inventory policies have a significant impact on any business's profit. The ultimate objective of inventory policies is to maximise profits and minimise associated costs. In the present competitive market scenario, every firm wants an increment in the demand for the product to maximise its profit. Therefore, for this purpose, they advertise the product in the potential marketplace. Bhunia and Maiti (1998) used a linear demand function depending on time, price, and advertisements in well-known media, and Mondal et al. (2009) used demand proportional to price and advertisement. Ghoreishi et al. (2013) discussed product returns in the joint optimisation of inventory policies. In recent years, no author has used demand-dependent advertisements in the media and product returns with deteriorating items simultaneously. Therefore, to express the scenario an inventory model that considers demand varies with the number of advertisements and deterioration with product returns is discussed. The scenario of shortages is also incorporated while the unsatisfied demand is partially backlogged.

1.2 Literature review

Pricing and inventory management are two critical components of the success of any business. The relevance of these aspects increases as the products start to deteriorate. In this area, many inventory models considered deterioration. Deterioration is referred to as decay, evaporation, spoilage, damage, or loss of marginal value, which affects product utilisation. After a certain period, every physical good started to lose its usefulness, such as food, medicine, liquids, and many more (Tiwari et al., 2016). For a deteriorating item with trade credit, Shaikh et al. (2018)developed a fuzzy inventory model. They assumed that demand is influenced by the selling price and the frequency of advertisements, and they analysed shortages with partial backlogs where shortages follow inventory (SFI) policy. In addition, Mishra et al. (2020) presented a sustainable inventory model with a controllable non-instantaneous deterioration rate and Adak and Mahapatra (2020) presented a multi-product inventory model where deterioration is a function of time and reliability. An EOQ model with a Weibull deterioration rate under a hybrid prepayment scheme is discussed by Alshanbari et al. (2021).

Joint pricing and inventory management concerns are connected to the degradation of goods, which many researchers have widely reviewed. Initially, Ghare and Shrader (1963) attempted to establish optimal ordering practices for declining products. They discussed an ordering policy based on a deteriorating inventory system with an exponential deterioration rate. Later, Tripathi and Chaudhary (2017) and Patro et al. (2017) adopted an EOQ model that rested on the variable deteriorating rate method to derive an inventory model that follows the Weibull distribution. Mahmoodi (2019) studied the joint pricing and ordering policy model for deteriorating items tossed around linear demand. Chen and Teng (2015) staged an inventory model conditioned on a linearly time-dependent decay rate and credit period-dependent constant demand rate. Jaggi et al. (2017) addressed a modelling approach for declining inventory products based on time-dependent demand without shortages. Singh et al. (2018) developed a model with a time-varying deterioration rate in which shortages are allowed. Chowdhury et al. (2015) discussed an inventory model in which the deterioration rate was constant, and demand was proportional to stock level and price. Later, a comprehensive review of deterioration inventory literature was provided by Janssen et al. (2016). Jaggi et al. (2019) constructed an inventory system with deterioration and stock-induced demand. Chakraborty et al. (2018) considered a deterioration rate that follows Weibull distribution, and the inventory system consists of ramp type demand pattern. Later, Sharma and Saraswat (2021) compared two inventory models with instant and delayed deterioration rates considering demand function that is price and time-dependent.

Abad (2001) conferred inventory lot-size problems based on partially backlogged shortages of perishable goods with variable deterioration rates. Later, Patel (2019) presented an inventory model for perishable products, with partial backlogging where demand varies with price and time. Rezagholifam et al. (2020) discussed an inventory strategy for non-instantaneous decaying products considering a price and stock-dependent demand function and Yang (2018) considered partial backlogging depending on time. Tiwari et al. (2018) addressed a standard inventory supply chain model with declining inventory products with documented demand as a dependent of price and partial backlogging. They also established an inventory policy with shortages for products with an expiration date. Maihami and Nakhai (2012) and Mukhopadhyay (2019) proposed an ordering and pricing strategy for the deterioration of inventories with the partial backlog of shortages where demand was dependent on time and price. Pervin et al. (2018) address the stochastic deterioration in the inventory system while considering the time demand function. Patro et al. (2018) created a crisp and fuzzy EOQ model that takes into account deterioration with imperfect quality items as well as the impact of learning on holding costs, ordering costs, and the number of imperfect quality items in each batch. They assumed that all products received were not in perfect condition, and after a screening process, defective products were removed from the inventory and sold at a discounted price. Also, Shaikh et al. (2019) discussed demand as a function of inventory level and partial backlogging to address the economic order quantity model. Recently, Geetha and Udayakumar (2020) has studied two distinct models: shortages are not permitted, and shortages are allowed with the partial backlog. Application of IT in managing inventory is well known (Singh and Prabhakar, 2021) and is useful in such cases also.

In the current competitive market scenario, the frequency of advertisement has significantly impacted the demand pattern of the product amongst the public. Here, the promotion in media or through word of mouth plays a vital role. The promotion and canvasing of a product by advertisements on available platforms such as magazines, newspapers, radio, television, movies, and more significant impact customers. Therefore, advertisement by any means is one of the most efficient approaches to spread the word about a product's popularity across all classes of consumers. As a result, this has a significant direct influence on increasing product demand (Al-Amin Khan et al., 2020).In addition, the unit selling price plays a vital role in opting for an item to use. Also, some specific items have seasonal demand, change with time. As a result, the demand rate is characterised as a variable of time, unit selling price, and advertisements all at the same time. Bhunia and Maiti (1998) demonstrated a deteriorating inventory system in which a product's demand rate is a function of time, selling price, and advertisements in media, with shortages and shortages partially back logged within the inventory cycle time. Under the hybrid permissible delay in payment policy, Shaikh (2017) proposed an inventory model for a deteriorating product while demand is depending on selling price and frequency of advertisements. In this direction, few researchers discussed inventory model considering demand varying with the frequency of advertisements such as Al-Amin Khan et al. (2020) for perishable products with advance payment, Kumar (2021) for decaying products with preservation technology, Rapolu and Kandpal (2019) for non-instantaneous deteriorating products under preservation technology and San-José et al. (2021) for lot-size inventory problem with holding cost is a potential function of time.

In the inventory system, vendors can send back all or a fraction of products that are not sold at the termination of the planning period to the producer/distributor and collect a partial/full refund as per the producer policy. So, product returns is an important term to be considered in inventory problems. Effective inventory management improves the returns process and allows businesses to manage the customer experience aspect effectively. Hess and Mayhew (1997) addressed the product returns issue on a broad direct market, and the problem was formulated using regression methods. They showed that the product returns rate rises with a rise in the product's price. Similarly, Anderson et al. (2008) discussed the substantially linear relationship between the number of sold items and the number of product returns. Chen and Bell (2009) assumed product returns as a function of price and product demand. Later, Zhu (2012) proposed aperiodic review model for a single product to optimise the pricing strategy with product returns.

The processing of returned items is handled by inventory management so that re-manufactured products can satisfy a portion of the customer's requirements. As a result, the companies involved in the supply chain operation must assess the influence of returned products on their inventory policy to have an effective inventory management system. Product return is a major problem owing to the unpredictability involved with the pricing, demand, and product quality. Thus, to tackle the problem companies must increase their information transparency to monitor the product returns behaviour of the end customers (Ambilkar et al., 2021). Parvini et al. (2014) created a two-echelon inventory model for a single reusable product, in which return rates are directly dependent on demand. Re-manufacturing is the process of transforming a damaged product into one that can be used again. They expanded the basic inventory model to incorporate manufacturing and re-manufacturing activities using a continuous review inventory policy and then solved the model using a dynamic programming approach. The inventory replenishment issues are also addressed with product returns in the inventory models. Although, on joint optimum inventory issues, several scholars have investigated the case of product returns. Many research papers considered the effect of product returns on inventory control, and their pricing policy assumes that the product returns are a function of the product's price and demand.

1.3 Contribution of the paper

In this paper, customer returns on the inventory system with deteriorating products under partial backlogging have been discussed. The multi-variate demand depending on unit selling prices, time, and the number of advertisements has been considered. Not a single researcher has worked on the notion of customer returns with multi-variate demand function depending on advertisements and shortage under deterioration to the best of our knowledge. The key contribution of the paper is to underwrite to fill the gap. For this, an inventory model for a decaying item is established under the scenario of product returns. The product returns are proportional to the increased quantity sold during the inventory planning cycle and the product's unit price. The shortcoming of products is backlogged but partially. An optimisation procedure for the proposed inventory model is perused to obtain the optimal time, replenishments, and selling price to derive maximum profit. Finally, a solution is presented, and a numerical example is solved to illustrate the proposed model.

The remaining sections of this paper are prearranged as follows. The next part outlines the assumptions and notations used to formulate the model in this study. In the third section, the mathematical model formulation and analysis for finding the optimal solution are presented. In the subsequent fourth section, we substantiate that an optimal solution exists for the objective function with a unique selling price value. Also, a solution algorithm to compute the optimum value of the optimal selling price, time without any deterioration is illustrated. Further, the fifth section consists of a numerical illustration to validate the solution procedure discussed, and sensitivity analysis is carried out for model parameters. In addition, the concavity for the selling price and time with total profit is presented. Finally, we make concluding remarks and include some recommendations for potential research in the future in the sixth section.

2 Assumption and notation

2.1 Notations

The notations utilised to formulate the problem are as below.

Parameters	
C	The purchase cost of every unit
A	The ordering cost of every order placed
h	Holding cost for every unit per unit time
l	Lost sale cost of every unit
s	Backorder cost for every unit per unit time
D	The demand of the product for the replenishment cycle
a	Minimum demand
b	Scale parameter for demand
c	Scale parameter for demand
N	Number of advertisements
α	Scale parameter for advertisement
Q	Order quantity
λ	Constant deterioration rate
β	The fraction of shortage back ordered
η	Fractional constant for advertising cost
δ	A positive constant for backorder
μ	A positive constant for product returns
SV	Salvage value per unit
Q^*	Optimised quantity per order
t_1^*	Optimal time without any shortages
p^*	The optimal value of selling price for a unit
I_1	Inventory level for time $t \in [0, t_1]$
I_2	Inventory level for time $t \in [t_1, T]$
I_0	Initial inventory level
S	The upper limit of demand backlogged
T	Replenishment cycle time length
TP (p, t_1)	Value of total profit over the replenishment cycle
Decision variables	
t_1	Time length without any inventory shortages
p	The selling price of one unit, where $p > C$

2.2 Assumptions

In this section, the subsequent assumptions are considered to design the inventory model.

a The rate of replenishment is sufficient to fulfill the order in time where the lead time is negligible.

b The determination of on-hand inventory is considered as a constant function with deteriorated inventory items.

c The demand rate of the products is proportional to the unit selling price p, time t, and the frequency of the advertisement, i.e., $D(p, t, N) = (a - b \times p + c \times t)N^{\alpha}$ where a, b, c, and α are non-negative (Bhunia and Maiti, 1998).

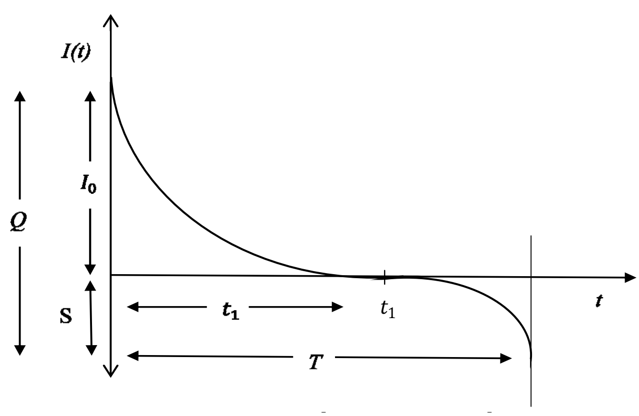
d The advertisement frequency is N in the replenishment cycle T, and the upper limit of the advertisement cost for a replenishment cycle is fixed.

e The time horizon is finite.

f Shortages are permitted, but the shortages that occurred are backlogged partially, and the shortages are back-ordered as $\beta(x) = k_0 \times e^{(-\delta x)}$, $(\delta > 0, 0 < k_0 \leq 1)$ and $0 \leq \beta(x) \leq 1$, $\beta(0) = 1$, where x is a waiting time for the subsequent order(Maihami and Nakhai, 2012).

g Considering the experimental results of Ghoreishi et al. (2013), in the inventory modelling, we have undertaken that the product returns increase with the selling price and the quantity sold together using the following linear equation: $R(p, t) = \beta \times p + \mu \times D(p, t, N)$, $(0 \leq \mu < 1$, and $\beta(x) \geq 0)$.

Figure 1 The interpretation of inventory levels graphically



3 Mathematical formulation

In the model formulation, the inventory model discussed by Yang et al. (2009) and later Maihami and Nakhai (2012) is adopted. According to the model concerned, the inventory system has different inventory levels at different points in time. The maximum initial inventory level at the opening of every inventory cycle is denoted as I_0. The inventory level drops to zero at the conclusion of each cycle due to deterioration and demand fulfillment where time $t \in [0, T]$. Finally, shortages occur during the planning period $[t_1, T]$ and therefore partial backlogging. Figure 1 represents the visual interpretation of different inventory levels.

The following equations provide the mathematical formulations for inventory levels:

The inventory levels throughout the time interval $(0, t_1)$ are represented by the differential equation.

$$\frac{dI_1(t)}{dt} + \lambda \times I_1(t) = -D(p,t,N) \qquad 0 \le t \le t_1 \tag{1}$$

while at time the inventory level decreases to zero, i.e., $I_1(t_1)=0$, during the time $(0, t)$ the inventory level is expressed as in equation (2):

$$I_1(t) = -\frac{N^{\alpha}}{\lambda}\left[\left(a - b\times p + c\times t - \frac{c}{\lambda}\right) - e^{\lambda(t_1 - t)}\left(a - b\times p + c\times t_1 - \frac{c}{\lambda}\right)\right], \quad 0 \le t \le t_1 \tag{2}$$

Also, at the initial phase of the inventory cycle at the time $t = 0$, the initial inventory (i.e., $I_1(0) = I_0$) is expressed as in the given equation (3):

$$I_0 = -\frac{N^{\alpha}}{\lambda}\left[\left(a - b\times p - \frac{c}{\lambda}\right) - e^{\lambda t_1}\left(a - b\times p + c\times t_1 - \frac{c}{\lambda}\right)\right] \tag{3}$$

The second time interval (t_1, T), showing partially backlogged shortages according to the fraction $\beta(T-t)$. Hence, the inventory level at time t is expressed as in equation (4):

$$\frac{dI_2(t)}{dt} = -\beta(T-t)\times D(p,t,N) \qquad t_1 \le t \le T$$

$$\Rightarrow \frac{dI_2(t)}{dt} = \frac{-D(p,t,N)}{e^{\delta(T-t)}} \qquad t_1 \le t \le T \tag{4}$$

At time t_1, the inventory level becomes zero; therefore, applying the boundary condition, $I_2(t_1) = 0$ the inventory I_2(t) is showed as in equation (5):

$$I_2(t) = -\frac{N^{\alpha}}{\delta}\begin{bmatrix} e^{-\delta(T-t)}\left(a - b\times p + c\times t - \dfrac{c}{\delta}\right) \\ +e^{-\delta(T-t_1)}\left(a - b\times p + c\times t_1 - \dfrac{c}{\delta}\right)\end{bmatrix}, \quad t_1 \le t \le T \tag{5}$$

After finding all the expressions for inventory level, by putting $t = T$ in the equation (5), the maximum quantity of demand backlogged (S) is presented in equation (6):

$$S = -I_2(T) = \frac{N^{\alpha}}{\delta}\left[\left(a - b\times p + c\times T - \frac{c}{\delta}\right) + e^{-\delta(T-t_1)}\left(a - b\times p + c\times t_1 - \frac{c}{\delta}\right)\right] \tag{6}$$

The total ordered amount in each inventory cycle (i.e., Q) is the sum of the maximum quantity of demand backlogged S. and initial inventory I_0, that is $Q = S + I_0$ and expressed as:

$$Q = \frac{N^{\alpha}}{\delta}\left[\left(a - b\times p + c\times T - \frac{c}{\delta}\right) + e^{-\delta(T - t_1)}\left(a - b\times p + c\times t_1 - \frac{c}{\delta}\right)\right] - \frac{N^{\alpha}}{\lambda}\left[\left(a - b\times p - \frac{c}{\lambda}\right) - e^{\lambda t_1}\left(a - b\times p + c\times t_1 - \frac{c}{\lambda}\right)\right] \tag{7}$$

Thus, in the whole inventory planning cycle, the different inventory costs and produced sales revenue are provided under the following aspects as in the following expressions:

a The cost for placing an order in the inventory cycle is fixed, as it was at the starting of the cycle, which is denoted by *A*.

b The inventory occurs in the period (0, t_1) as shown in Figure 1. Therefore, the inventory holding cost (HC) occurs only in this period (0, t_1) and computed as below

$$HC = h\int_0^{t_1} I_1(t)dt \tag{8}$$

$$HC = -h\times\frac{N^{\alpha}}{\lambda}\left[\left(a - b\times p + \frac{c}{2}t_1 - \frac{c}{\lambda}\right)t_1 + \frac{\left(1 - e^{\lambda t_1}\right)}{\lambda}\left(a - b\times p + c\times t_1 - \frac{c}{\lambda}\right)\right] \tag{9}$$

c During the time period (t_1, *T*), shortages occur, and the shortage cost (SC) due to backlog is estimated as

$$SC = s\int_{t_1}^{T} -I_2(t)dt \tag{10}$$

$$SC = s\times\frac{N^{\alpha}}{\delta}\left[\begin{array}{l}\dfrac{a - b\times p + cT}{\delta} - \dfrac{2c}{\delta^2} - e^{-\delta(T - t_1)}\dfrac{(a - b\times p + c\times t_1)}{\delta} \\ + \dfrac{2c}{\delta^2}\times e^{-\delta(T - t_1)} + \left(a - b\times p + c\times t_1 - \dfrac{c}{\delta}\right)e^{-\delta(T - t_1)}(T - t_1)\end{array}\right] \tag{11}$$

d Due to the lost sales, the opportunity cost (OC) is expressed below in equation (13)

$$OC = l\int_{t_1}^{T} D(p,t,N)\left(1 - \beta(T - t)\right)dt \tag{12}$$

$$OC = lN^{\alpha}\left[\begin{array}{l}(T - t_1)\left\{a - b\times p + \dfrac{c}{2}(T + t_1)\right\} \\ + \dfrac{1}{\delta}\left\{\left(a - b\times p + c\times T + \dfrac{c}{\delta}\right) - e^{-\delta(T - t_1)}\left(a - b\times p + c\times t_1 + \dfrac{c}{\delta}\right)\right\}\end{array}\right] \tag{13}$$

e The value of purchase cost (PC) is

$$PC = c(I_0 + S) \tag{14}$$

$$PC = C\frac{N^{\alpha}}{\delta}\left[\left(a - b\times p + c\times T - \frac{c}{\delta}\right) + e^{-\delta(T - t_1)}\left(a - b\times p + c\times t_1 - \frac{c}{\delta}\right)\right] - C\frac{N^{\alpha}}{\lambda}\left[\left(a - b\times p - \frac{c}{\lambda}\right) - e^{\lambda t_1}\left(a - b\times p + c\times t_1 - \frac{c}{\lambda}\right)\right] \tag{15}$$

f The value of return cost (RC) is

$$RC = (p - SV)\int_0^{t_1}\left(\mu\times D(p,t,N) + \beta\times p\right)dt \tag{16}$$

$$RC = (p - SV)\left[\mu\times N^{\alpha}\left(a - b\times p + \frac{c}{2}t_1\right)t_1 + \beta\times p\times t_1\right] \tag{17}$$

g The value advertisement cost (AC) is

$$AC = \eta\times p\times N\int_0^{t_1} D(p,t,N)dt \tag{18}$$

$$AC = \eta\times p\times N^{\alpha+1}\left(a - b\times p + \frac{c}{2}t_1\right)t_1 \tag{19}$$

h The expression of sales revenue (SR) generated from the sales during the inventory cycle is

$$SR = p\times\left(\int_0^{t_1}\alpha\times D(p,t,N)\,dt + S\right) \tag{20}$$

$$SR = p\times N^{\alpha}\left(a - b\times p + \frac{c}{2}t_1\right)t_1 + p\frac{N^{\alpha}}{\delta}\left[\begin{array}{l}\left(a - b\times p + c\times T - \frac{c}{\delta}\right)\\ +e^{-\delta(T - t_1)}\left(a - b\times p + c\times t_1 - \frac{c}{\delta}\right)\end{array}\right] \tag{21}$$

As a result, the value of total profit per unit time (written in notation as $TP(p, t_1)$ is given in equation (22) below:

$$TP(p,t_1) = SR - A - HC - SC - AC - RC - OC - PC \tag{22}$$

The purpose of the inventory model is to compute the selling price of each unit p and the time t_1 until there are no shortages occurs that maximise $TP(p, t_1)$with restrictions $p > 0$ and $0 < t_1 < T$, where p and t_1 are continuous variables. The necessary conditions for equation (23) to calculate p^* and t_1^* are given in the upcoming Section 4:

$$TP(p,t_1)=\begin{bmatrix} p\times\left(\int\limits_0^{t_1}(\alpha\times D\times(p,t,N))\,dt+S\right)-A-h\int\limits_0^{t_1}I_1(t)\,dt-s\int\limits_{t_1}^{T}I_2(t)dt \\ -l\int\limits_{t_1}^{T}\left(D(p,t,N)(1-\beta(T-t))\right)dt-c\left(I_0+S\right) \\ -(p-SV)\int\limits_0^{t_1}(\alpha\times D(p,t,N)+\beta\times p)\,dt-\eta\times p\times N\int\limits_0^{t_1}D(p,t,N)dt \end{bmatrix} \tag{23}$$

$$\begin{aligned} &= p\times N^{\alpha}\left(a-b\times p+\frac{c}{2}t_1\right)t_1+p\times\frac{N^{\alpha}}{\delta}\begin{bmatrix}\left(a-b\times p+c\times T-\frac{c}{\delta}\right)\\ +e^{-\delta\left(T-t_1\right)}\left(a-b\times p+cvt_1-\frac{c}{\delta}\right)\end{bmatrix}\\ &-A+h\times\frac{N^{\alpha}}{\lambda}\left[\left(a-bvp+\frac{c}{2}\times t_1-\frac{c}{\lambda}\right)t_1+\frac{\left(1-e^{\lambda t_1}\right)}{\lambda}\left(a-b\times p+c\times t_1-\frac{c}{\lambda}\right)\right]\\ &-s\times\frac{N^{\alpha}}{\delta}\begin{bmatrix}\frac{a-b\times p+c\times T}{\delta}-\frac{2c}{\delta^2}-e^{-\delta\left(T-t_1\right)}\frac{\left(a-b\times p+c\times t_1\right)}{\delta}\\ +\frac{2c}{\delta^2}\times e^{-\delta\left(T-t_1\right)}+\left(a-b\times p+c\times t_1-\frac{c}{\delta}\right)e^{-\delta\left(T-t_1\right)}\left(T-t_1\right)\end{bmatrix}\\ &-l\times N^{\alpha}\begin{bmatrix}\left(a-b\times p+\frac{c}{2}\left(T+t_1\right)\right\}\left(T-t_1\right)\\ +\frac{1}{\delta}\left\{\left(a-b\times p+c\times T+\frac{c}{\delta}\right)-e^{-\delta\left(T-t_1\right)}\left(a-b\times p+c\times t_1+\right)\frac{c}{\delta}\right\}\end{bmatrix}\\ &-C\times\frac{N^{\alpha}}{\delta}\left[\left(a-b\times p+c\times T-\frac{c}{\delta}\right)+e^{-\delta\left(T-t_1\right)}\left(a-b\times p+c\times t_1-\frac{c}{\delta}\right)\right]\\ &+C\times\frac{N^{\alpha}}{\lambda}\left[\left(a-b\times p-\frac{c}{\lambda}\right)-e^{\lambda t_1}\left(a-b\times p+c\times t_1-\frac{c}{\lambda}\right)\right]\\ &-(p-SV)\left[\mu\times N^{\alpha}\left(a-b\times p+\frac{c}{2}\times t_1\right)t_1+\beta\times p\times t_1\right]\\ &-\eta\times p\times N^{\alpha+1}\left(a-b\times p+\frac{c}{2}\times t_1\right)t_1 \end{aligned} \tag{24}$$

4 Results and analysis

4.1 Theoretical results

The inventory model's purpose is to determine the best inventory strategy for maximising the system's total profit. In order to accomplish the goal, there exists a unique p, i.e., p^* for any given p, maximises the objective of the problem. And also, for every inventory cycle, every given t_1, there is a discrete p^* that produces the maximum total profit.

The total profit $TP(p, t_1)$expression is a function of selling price p and time t_1 till then, no deterioration occurs. As a result, $\frac{\partial TP(p,t_1)}{\partial t_1}=0$ and $\frac{\partial TP(p,t_1)}{\partial t}=0$ are required conditions to get the highest total profit per unit time for every given value of p [as in equation (24)]. Therefore

$$
\begin{aligned}
\frac{\partial TP(p,t_1)}{\partial t_1} &= p\times N^{\alpha}\left[\left(a-b\times p+2c\times t_1\right)+\left(a-b\times p+c\times t_1\right)\right] \\
&-0+h\times\frac{N^{\alpha}}{\lambda}\left[\left(a-b\times p+c\times t_1\right)\left(1-e^{\lambda t_1}\right)\right] \\
&-s\times\frac{N^{\alpha}}{\delta}\left[e^{-\delta(T-t_1)}\left\{\frac{2c}{\delta}-\left(a-b\times p+c\times t_1\right)\right\}\right. \\
&\left.+e^{-\delta(T-t_1)}\left\{\left(a-b\times p+c\times t_1-\frac{c}{\delta}\right)\left(\delta\left(T-t_1\right)-1\right)+c\left(T-t_1\right)\right\}\right] \\
&+l\times N^{\alpha}\left[\left(a-b\times p+c\times t_1\right)-e^{-\delta(T-t_1)}\left(a-b\times p+c\times t_1+\frac{2c}{\delta}\right)\right] \\
&-\eta\times p\times N^{\alpha+1}\left(a-b\times p+c\times t_1\right)-C\times N^{\alpha} \\
&\left[\left(a-b\times p+c\times t_1-\frac{c}{\lambda}\right)e^{-\delta(T-t_1)}-\left\{a-b\times p+c\times t_1+c\left(\frac{1}{\lambda}-\frac{1}{\delta}\right)\right\}e^{\lambda t_1}\right] \\
&-(p-SV)\left[\mu\times N^{\alpha}\left(a-b\times p+c\times t_1\right)+\beta\times p\right] = 0
\end{aligned}
\tag{25}
$$

$$
\begin{aligned}
\frac{\partial^2 TP\left(p,t_1\right)}{\partial t_1^{\,2}} &= p\times N^{\alpha}[3c]+h\times\frac{N^{\alpha}}{\lambda}\left[c\left(1-e^{\lambda t_1}\right)-\lambda\times e^{\lambda t_1}\left(a-b\times p+c\times t_1\right)\right] \\
&-(p-SV)\left[\mu\times N^{\alpha}\times c\right]-s\times N^{\alpha} \\
&\begin{bmatrix} e^{-\delta(T-t_1)}\frac{c}{\delta}-2\left(a-b\times p+c\times t_1\right)e^{-\delta(T-t_1)}+c\left(T-t_1-\frac{1}{\delta}\right) \\ +e^{-\delta(T-t_1)}\left\{\left(a-b\times p+c\times t_1-\frac{c}{\delta}\right)\left(\delta\left(T-t_1\right)-1\right)+c\left(T-t_1\right)\right\} \end{bmatrix} \\
&+l\times N^{\alpha}\left[c-\delta\times e^{-\delta(T-t_1)}\left(a-b\times p+c\times t_1+\frac{2c}{\delta}\right)-c\times e^{-\delta(T-t_1)}\right] \\
&-\eta\times p\times N^{\alpha+1}\times c-C\times N^{\alpha}\begin{bmatrix} \delta\left(a-b\times p+c\times t_1-\frac{c}{\lambda}\right)e^{-\delta(T-t_1)} \\ +c\times e^{-\delta(T-t_1)} \\ -\lambda\begin{Bmatrix} a-b\times p+c\times t_1 \\ +c\left(\frac{1}{\lambda}-\frac{1}{\delta}\right)\end{Bmatrix}e^{\lambda t_1}-c\times e^{\lambda t_1} \end{bmatrix}
\end{aligned}
\tag{26}
$$

From equations (25) and (27), the optimal values of p and t_1 are computed by putting the values of all the given parameters simultaneously.

Henceforth, the condition under which the optimal and unique selling price exists is addressed subsequently. The necessary condition for total profit $TP(p, t_1)$ to produce maximum profit is $\frac{\partial TP(p,\ t_1)}{\partial p} = 0$ for every optimal value t_1 obtained from equations (25) and (27); that is

$$\begin{aligned}\frac{\partial TP(p,t_1)}{\partial p} &= -b\times p\times N^{\alpha}\left[t_1+\frac{1}{\delta}+\frac{e^{-\delta(T-t_1)}}{\delta}\right] \\ &+c\times N^{\alpha}\left[\frac{b}{\lambda}\left(1-e^{\lambda t_1}\right)+\frac{b}{\delta}\left(1+e^{-\delta(T-t_1)}\right)\right] \\ &+N^{\alpha}\left[\left(a-b\times p+\frac{c}{2}\times t_1\right)t_1+\frac{1}{\delta}\left\{\begin{array}{l}\left(a-b\times p+c\times T-\frac{c}{\delta}\right) \\ +e^{-\delta(T-t_1)}\left(\begin{array}{l}a-b\times p \\ +c\times t_1-\frac{c}{\delta}\end{array}\right)\end{array}\right\}\right] \\ &+s\times b\times\frac{N^{\alpha}}{\delta^2}\left[1+\frac{e^{-\delta(T-t_1)}}{\delta}+e^{-\delta(T-t_1)}\left(T-t_1\right)\right] \\ &+b\times l\times N^{\alpha}\left[\left(T-t_1\right)+\frac{1}{\delta}+e^{-\delta(T-t_1)}\right]-h\times\frac{N^{\alpha}}{\lambda}\left[b-\frac{b}{\lambda}\left(1-e^{\lambda t_1}\right)\right] \\ &-(p-SV)\left(-\mu\times N^{\alpha}\times b\times t_1+\beta\times t_1\right) \\ &-\left(\mu\times N^{\alpha}\left(a-b\times p+\frac{c}{2}\times t_1\right)t_1+\beta\times p\times t_1\right) = 0\end{aligned} \tag{27}$$

The above equation (27) (i.e., $\frac{\partial TP(p,t_1)}{\partial p}=0$) has a solution optimal solution for p. Further, to obtain the optimal value of p the second-order derivations of total profit $\left(TP(p,t_1)\right)$ with respect to p must produce negative value, and the expression of the second derivative is as follows

$$\begin{aligned}\frac{\partial^2 TP(p,t_1)}{\partial p^2} &= -b\times N^{\alpha}\left[t_1+\frac{1}{\delta}+\frac{e^{-\delta(T-t_1)}}{\delta}\right]-b\times N^{\alpha}\left[t_1+\frac{1}{\delta}\left\{1+e^{-\delta(T-t_1)}\right\}\right] \\ &-\left(\mu\times N^{\alpha}\times b\times t_1-\beta\times t_1\right)+\left(\mu\times N^{\alpha}\times b\times t_1-\beta\times t_1\right) \le 0\end{aligned} \tag{28}$$

Further, there exists a distinct value of p (i.e., p^*) for every t_1^*, with the help of p^* the maximum total profit per unit time is obtained provided by having values of other parameters.

In conclusion, the total profit expression is a concave function (depict a concave shape to obtain a maximum value) of p for a given t_1^* shown in Figure 2, hence the optimum p derived from equation (27) is unique, which produces the maximum total profit for the concerned problem. Furthermore, an optimal solution procedure to solve the problem is proposed in the next section by concluding the above results.

4.2 Solution procedure

The total profit function $TP(p, t_1)$ consists of two variables: t_1 the time interval without any shortage and p the selling price of a unit. The subsequent solution procedure is proposed to obtain the optimal solution to the problem. To find the optimal values of p, t_1 and total profit $TP(p, t_1)$, the following procedure is applied.

Step 1 Find the first-order derivatives of total profit function with respect to selling price and time without any shortage, i.e.,

$$\frac{\partial TP(p,t_1)}{\partial p}$$

and

$$\frac{\partial TP(p,t_1)}{\partial t_1}.$$

Step 2 Let the p^* and t_1^* be the values satisfying the equations

$$\frac{\partial TP(p,t_1)}{\partial p}=0$$

and

$$\frac{\partial TP(p,t_1)}{\partial t_1}=0.$$

Step 3 Find the second-order derivative for $TP(p, t_1)$ concerning p and t_1 i.e.,

$$\left(\frac{\partial^2 TP(p,t_1)}{\partial p^2}\right)_{(p^*,t_1^*)}$$

$$\left(\frac{\partial^2 TP(p,t_1)}{\partial t_1^2}\right)_{(p^*,t_1^*)} \textit{and} \left(\frac{\partial^2 TP(p,t_1)}{\partial p \partial t_1}\right)_{(p^*,t_1^*)} \tag{29}$$

Step 4 If the objective is a concave function, then the necessary and sufficient conditions for the maximum profit function are

$$\left(\frac{\partial^2 TP(p,t_1)}{\partial p \partial t_1}\right)_{(p^*,t_1^*)} - \left(\frac{\partial^2 TP(p,t_1)}{\partial p^2}\right)_{(p^*,t_1^*)} \times \left(\frac{\partial^2 TP(p,t_1)}{\partial t_1^2}\right)_{(p^*,t_1^*)} < 0 \tag{30}$$

and any of the given conditions

$$\left(\frac{\partial^2 TP(p,t_1)}{\partial p^2}\right)_{(p^*,t_1^*)} < 0 \,or \left(\frac{\partial^2 TP(p,t_1)}{\partial t_1^2}\right)_{(p^*,t_1^*)} < 0 \tag{31}$$

Step 5 Find the optimal value of the profit function by putting values obtained from fulfilling the necessary and sufficient conditions in step 4.

Since $TP(p^*, t_1^*)$ is a complex exponential function; this is hard to express the validity of sufficient conditions analytically. Hence, the significance and validity of the above expressions in equation (30) and (31) is assessed numerically and presented graphically with Figure 2 in the subsequent Section 5.

5 Numerical illustration

5.1 Numerical example

The solution procedure is illustrated by applying the above-proposed solution method to compute the optimal values with the numerical example given below. The results are computed with the help of the solver available with Microsoft Excel and Lingo 14. The example consists of the following parameters, and Table 1 consists of the model parameters' value.

Table 1 Model parameters with their values

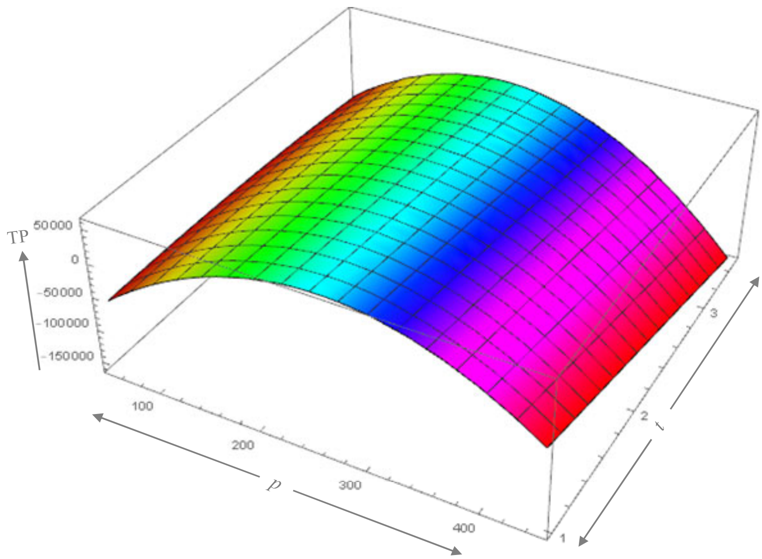
Parameter	*Detail*
C = \$80	Unit purchasing cost per unit
a = 200	Minimum demand
b = 0.5	Scale parameter for the demand
c = 10	Scale parameter for the demand
N = 6.0	Number of advertisements
α = 0.2	Scale parameter for the advertisement
β = 0.3	The fraction of shortage back ordered
h = \$20	Holding cost
s = \$25	Backorder cost
l = \$20	Cost of lost sale per unit
A = \$150	The ordering cost of every order placed
η = 0.03	Fractional constant for advertising cost
λ = 0.2	Constant deterioration rate
δ = 0.4	A positive constant for backorder
μ = 0.2	A positive constant for product returns
T = 6.0	Replenishment cycle time
SV = \$200	Salvage value per unit

Table 2 shows the optimal results of the decision variable as well as the total profit. In addition, Figure 2 depicts the total profit function's concavity. The concavity represents that there is a different optimal solution exists for the specified profit function $TP(p, t_1)$.

Table 2 optimal solution and optimal values

Variable	*Notation*	*Optimal value*
The optimal value of selling price for a unit	p^*	273,643
The optimal time without any shortages	t_1^*	2,535
The optimal total profit over the replenishment cycle	TP^*	35,237.657

Figure 2 The concavity of profit function $TP(p, t_1)$ (see online version for colours)



5.2 Sensitivity analysis

The impact of various parameters over total profit function and other optimal values are shown in the following tables. The sensitivity analysis is instigated by varying the parameters of demand (see Table 3); and parameters of back-ordering, advertising cost, and product returns (see Table 4).

Table 3 Sensitivity analysis of the demand function's parameters (i.e., *a, b, c, N* and α)

Parameter	*% Change*	*p*	t_1	$Q(p, t_1, N)$	$D(p, t_1, N)$	$TP(p, t_1)$
a	–20	243,534	2,074	468,696	144,269	14,821.711
	–10	258,523	2,302	610,481	205,008	23,865.702
	0	273,643	2,535	771,551	275,208	35,237.657
	10	288,903	2,782	956,144	356,127	49,081.504
	20	304,342	3,048	1170,200	449,599	65,569.317
b	–20	307,643	2,862	995,316	374,584	57,134.365
	–10	289,170	2,682	872,147	319,584	44,904.110
	0	273,643	2,535	771,551	275,208	35,237.657
	10	260,314	2,413	686,169	237,855	27,519.712
	20	248,762	2,308	611,669	205,657	21,316.334
c	–20	269,832	2,552	754,756	274,894	33,101.819
	–10	271,740	2,543	763,031	274,989	34,155.167
	0	273,643	2,535	771,551	275,208	35,237.657
	10	275,542	2,529	780,316	275,551	36,349.166
	20	277,436	2,524	789,335	276,023	37,489.589

Table 3 Sensitivity analysis of the demand function's parameters (i.e., *a, b, c, N* and α) (continued)

Parameter	*% Change*	*p*	t_1	$Q(p, t_1, N)$	$D(p, t_1, N)$	$TP(p,t_1)$
N	–20	270,429	2,747	803,722	295,190	35,750.238
	–10	272,018	2,645	789,387	286,110	35,542.144
	0	273,643	2,535	771,551	275,208	35,237.657
	10	275,341	2,419	750,484	262,539	34,869.416
	20	277,161	2,293	762,323	248,052	34,464.382
α	–20	271,675	2,464	710,577	251,036	31,762.881
	–10	272,660	2,500	740,448	262,877	33,460.784
	0	273,643	2,535	771,551	275,208	35,237.657
	10	274,623	2,571	803,939	288,051	37,096.656
	20	275,600	2,606	837,661	301,422	39,041.045

Table 4 Sensitivity analysis for the parameters of shortages, product returns, and advertisement (i.e., β, μ and η)

Parameter	*% Change*	*p*	t_1	$Q(p, t_1, N)$	$D(p, t_1, N)$	$TP(p, t_1)$
β	–20	279,681	2,755	796.993	291.416	38,596.170
	–10	276,514	2,639	783.568	283.045	36,839.942
	0	273,643	2,535	771.551	275.208	35,237.657
	10	271,024	2,441	760.673	267.833	33,768.007
	20	268,625	2,354	750,721	260,856	32,413,714
μ	–20	274,024	2,607	787,284	283,595	36,062.800
	–10	273,826	2,571	779,407	279,410	35,646.600
	0	273,643	2,535	771,551	275,208	35,237.657
	10	273,475	2,500	763,705	270,984	34,835.886
	20	273,324	2,464	755,858	266,732	34,441.207
η	–20	271,766	2,789	845,030	311,583	38,119.126
	–10	272,626	2,665	808,465	293,680	36,636.264
	0	273,643	2,535	771,551	275,208	35,237.657
	10	274,852	2,399	733,948	255,984	33,962.318
	20	276,310	2,253	695,141	235,720	32,706.420

Figure 3 represents the sensitivity for the total profit concerning parameters of demand which shows that *a* and α are more sensitive parameters.

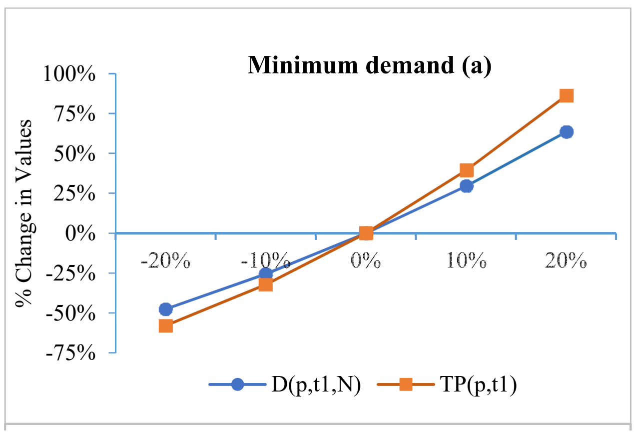
Table 3 may be used to make the following observations:

- If the parameters a (minimum demand), *c* (scale parameter), and α parameter for advertisement increase (decrease), then the order quantity and total profit function increase (decrease). Enlarged demand rises the order quantity is also increasing, which results in higher profit. This occurs due to a high flash in demand, which more suggests devoting more in the advertisement for better profit.
- If the parameters *b* (scale parameter) and parameter *N* for the number of advertisements increases (decreases), then the order quantity and total profit function decrease (increase). The demand showing a downward stream concerning the

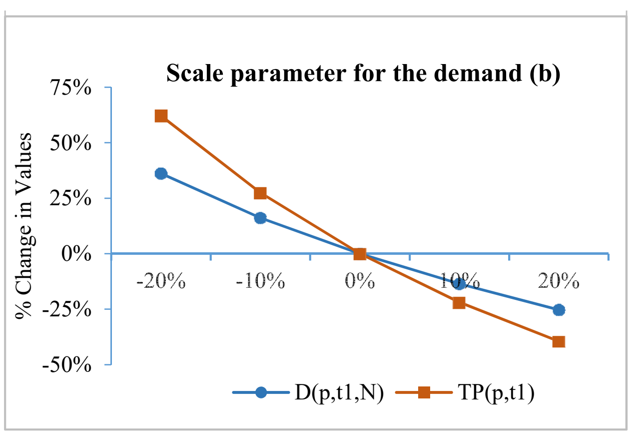
number of advertisements but the scale parameter presenting a positive impact on demand and total profit as well. The variation in the number of advertisements directly impacting the selling price and time. In the inventory model, the demand is depending on price and time along with advertisements. That is why demand is showing a downward graph with respect to parameter N. Thus, we can conclude that increasing the frequency of advertisements is not suitable under this scenario but the decision manager can increase the advertisement scale parameter to increase the demand and gain more profit under this proposed scenario of an inventory problem.

From Table 4, the observations can be carried out; if the parameters β (back-ordering parameter), η (advertising cost parameter), and μ parameter for product returns increases (decreases), then the order quantity and total profit function decrease (increase). In addition, while the demand is increasing the total profit is also increasing simultaneously.

Figure 3 The behaviour of demand parameters over total profit and demand (see online version for colours)

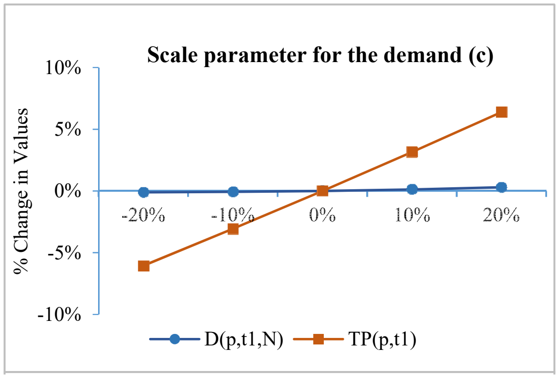


(a)

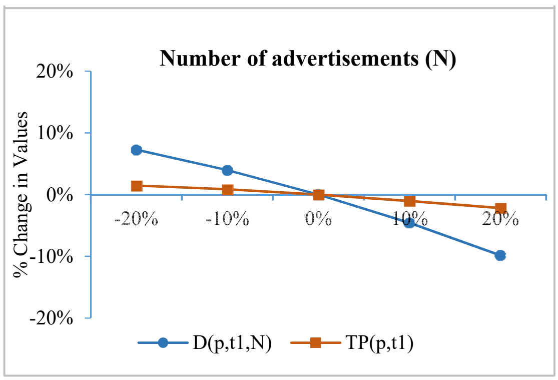


(b)

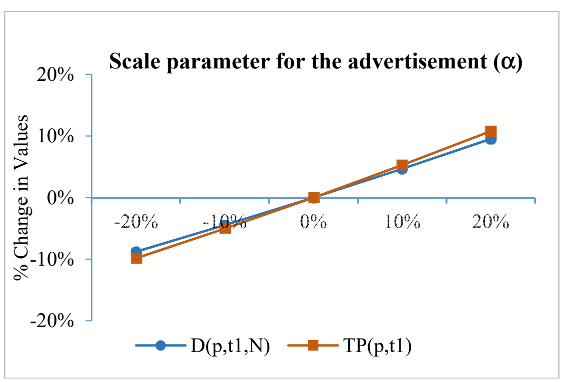
Figure 3 The behaviour of demand parameters over total profit and demand (continued) (see online version for colours)



(c)



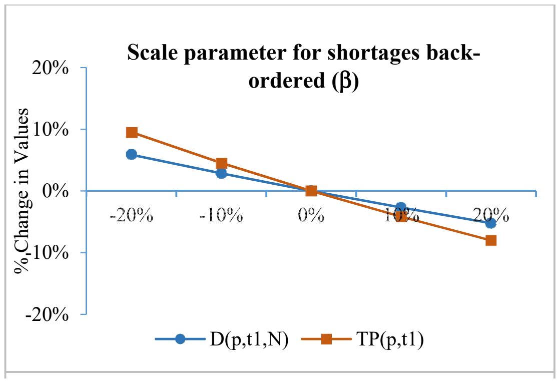
(d)



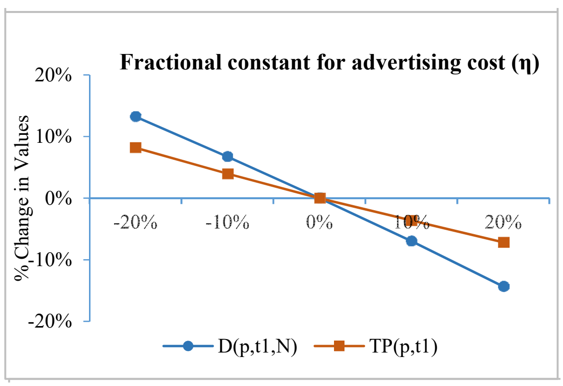
(e)

Figure 4 is representing the sensitivity for the total profit and demand concerning parameters of back-ordering, advertising cost, and product returns which shows that β and η are more sensitive parameters towards the total profit and demand. The parameters for product returns (μ) is very less sensitive towards demand and total profit with the considered range of % change. The graph shows that if the value of any one of the three-parameter is reduced then the profit for the inventory system in increases. Therefore, managers should reduce the shortages and product returns in order to gain more profit from the system. The analysis also suggesting the decision-maker to choose the advertisement medium with lessor advertisement cost.

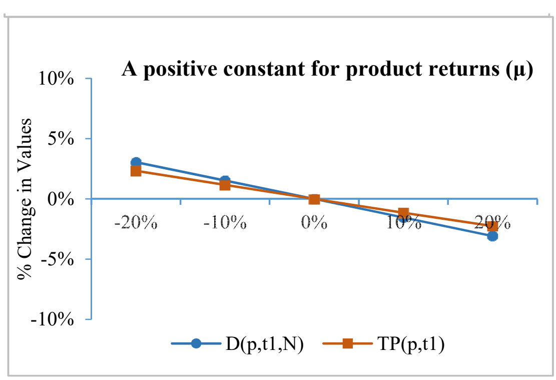
Figure 4 The impact of various parameters on total profit and demand function (see online version for colours)



(a)



(b)



(c)

6 Conclusions and future recommendations

An inventory model for deteriorating items with product returns has been presented in this study. With the growing complexity of inventory systems, it is of extreme significance to minimise the variance in demand and supply and balanced the shortages. In order to improve the total profit, an extra investment in advertisement to interest the more customers has been considered. Practically, the promotional effort is also essential to boost sales. To make the model more relevant and applicable in practice, the demand of the product is considered to be dependent on selling price, time, and advertisements. The proposed model allows shortages with partial backlogging and a constant deterioration rate. A suitable model for retailers to control their replenishment while shortages occurs and shortages are backlogged partially. The model assumed with exponential backlogging rate is depending on the total customers waiting for the upcoming order has been established. Following that, an effective solution technique is suggested for computing the optimal solution of the inventory problem. An example is provided to validate the solution technique and sensitivity testing of all parameters.

In a variety of ways, the suggested model can be reformed or expanded; for example, trade credit and time-dependent deterioration might be evaluated. Further, the demand can be considered as stock-dependent, environment-sensitive, or freshness or quality of the product. Also, the effect of imperfect quality items and inflation on economic policies can be considered. Finally, changes in optimal policies can be studied under the assumptions of carbon emission and preservation technology investment consideration.

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