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Effect of adaptation gain and reference model in MIT and Lyapunov rule–based model reference adaptive control for first- and second-order systems

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Abstract

An adaptive control law encompasses a regulating control rule compensating for system dynamics variations by adjusting the controller characteristics to maintain the overall system performance. Recently, some techniques have been developed based on fundamental aspects of the adaptability of living organisms. The adaptive control method is a technique that measures the dynamic characteristics of the plant automatically and continuously to make a comparison with its required output. It utilizes the difference between these two to commute adaptable system parameters to maintain optimal performance regardless of the system variations. The behavior of the adaptation rule is significantly affected by the adaptation gain value. Here, it has also been investigated that the adaptation gain range is wide for the systems with the lower order. The appropriate range of adaptation gain decreases as the order of the system increases. In the present work, the model reference adaptive control (MRAC) for first- and second-order systems has been designed and investigated using a wide range of values for the adaptation gain and variations in the reference model parameters. The MIT (Massachusetts Institute of Technology) and Lyapunov rules are applied for the analyses of systems. On the MATLAB/Simulink platform, all the adaptation process comparisons, variations, and investigations have been carried out by altering the adaptation gain and the reference model parameters. The obtained results present encouraging outcomes.

Keywords

MRAC, adaptation gain, reference model, MIT rule, Lyapunov rule, adaptation mechanism

Introduction

A control system is an interconnection of the physical components to produce a needed action while including the relevant control function. This definition applies to a control system in its broadest sense. Because engineering experiments require the correct operation of control systems, the practical implementation of modern control theory has been the subject of extensive research, precisely adaptive control (Dinakin and Oluseyi, 2021). Adaptive control is still developing in the current control methods, even though studies and development in this field go back a long time. In the 1950s, it was inspired by the issue of modeling an autopilot to operate an aircraft for an extended range of speeds and altitudes. The result was improved gain scaling based on supporting airspeed measurements. Kalman (Simon, 2010) developed a universal self-tuning regulator concept with a clear identity of the parameters of a single-input single-output (SISO) linear system. He has applied estimation of the parameter to bring up to date an optimal linear quadratic controller. The concept is further advanced by other investigators like Lyapunov (Banerjee et al., 2018) and Parks. Adaptive control concepts are currently present in areas where standard control systems may not effectively provide comfort due to the nature of the circumstance. These areas include things like (Swarnkar et al., 2010): inertia, loads, and other forces acting on the system vary severely; probability of unanticipated or regular disturbances; and probability of unexpected and random faults.

The traditional controllers with static gain value may not cope with the difficulties dissertated previously. For absolute adaptive behavior, exclusive adaptive control methods are required (Jain and Nigam, 2013; Neogi et al., 2018; Pankaj et al., 2011). Real-time updates to the control process coefficients are made using adaptive control for accounting for environmental and system changes. Along with the changes in the circumstances, it also affects the system's transfer function. Because of the complexity of the controller, a model reference adaptive control (MRAC) system may nearly always be deployed most effectively when using a digital computer. Among different types of adaptive control methods, this paper mainly discusses the MIT and Lyapunov rules of the

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MRAC scheme separately for first- and second-order systems. In MRAC (Mukherjee et al., 2018a; Patra, 2020; Sethi et al., 2017; Stellet, 2011), the output response is made to roughly trace the response of a reference model despite changes in the plant's parameters. This process is accomplished by forcing the output response to resemble the response of the reference model. The controller's parameters are changed to get the desired level of closed-loop performance. This process involves parameter estimation for the controller, which appropriately modifies the plant's transfer function. This estimation is done so that the plant's performance is the same as the reference models. The theory of augmented error (Aydin and Gurleyen, 2018; Mirkin, 2005; Monopoli, 1974) is one of the design strategies that may be implemented in an MRAC system, in addition to the MIT rule and the Lyapunov rule. In this paper, investigations of first- and second-order systems are carried out with the MIT rule and Lyapunov rule for designing a novel MRAC law. The system response has been figured out for various values of adaptation gain and also for different reference models. Finally, the significant contribution of the work may be listed as follows:

- MIT and Lyapunov-based MRAC controllers have been designed to trace the reference model for first- and second-order systems. How the adaptation gain affects the system's response and stability has also been investigated.
- In order to match the response with the reference model, a suitable range of adaptation gain for first- and second-order systems has been found.
- The effect of the reference model's parameters on the system response and the best way to pick a reference model for first- and second-order systems have been evaluated.
- It has been noted that for first-order system (46), the adaptation gain should be in the range of 1–1000, and the reference model's time constant should be greater than the time constant of first-order system (i.e. $a_m > a$).
- It has been observed for the second-order system (47) that, as the natural frequency of the reference model is increased, the settling time and overshoot of the system are decreased for the range of $\alpha = 1$–3 in both MIT and Lyapunov rules. But the system becomes unstable for $\alpha = 4$ and 5 with the MIT rule; with the Lyapunov rule for the same values of α, the system is stable with some oscillations.
- All the investigations are done for both the MIT rule and the Lyapunov rule separately, and comparative analysis suggests that the design using the Lyapunov rule performs superior to the MIT rule.

MRAC

MRAC is a form of adaptive control that belongs to the broader category of non-dual adaptive control (Král and Šimandl, 2008). A reference model may define the performance of the system. After comparing the actual output to the modeled output, the feedback controller settings are modified using a different approach. The MRAC is modeled to manipulate the plant or system output to track the reference model. The model reference adaptive system has two types of loops. First is an inner loop, also known as a regulator loop. It is a standard control loop that consists of a regulator and the plant to update the plant parameter through an adaptation mechanism. The second is the outer loop, also known as the adaptation loop. This loop coordinates the regulator parameters to manage the steady-state error between the system output and modeled output down to zero. The block diagram for MRAC is shown in Figure 1.

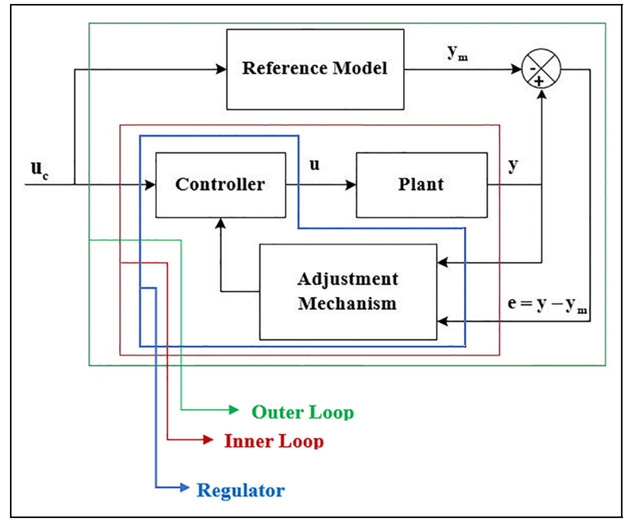

Figure 1. Block diagram of model reference adaptive controller.

Components of model reference adaptive controller

Reference model

The ideality of the response of the adaptive control system to the external commands is specified by the reference model. It reflects the performance specifications based on the control objectives. The ideality shown by the reference model should be such that it may be accessible to the MRAC system.

Controller

Commonly, the controller is designed taking account of various adaptable parameters. The controller rule is clearly described in this work by two parameters θ_1 and θ_2. Regarding the adjustable constraints, the controller design is linear (i.e. linear parameterization). Usually, to get an adaptation mechanism, linear parameterization requires an adaptive controller design with definite stability and tracking (Hosseini et al., 2021). The adaptation gain modifies the adaptation mechanism/control algorithm and largely determines the size of these control parameters. This modification occurs because the adaptation process is changed by adaptation gain.

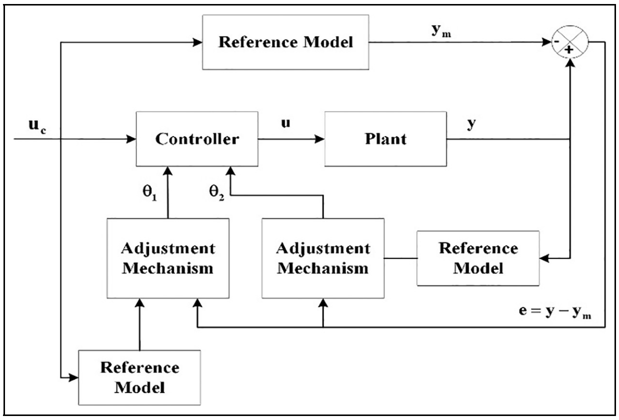

Figure 2. Block diagram model for MRAC using MIT rule.

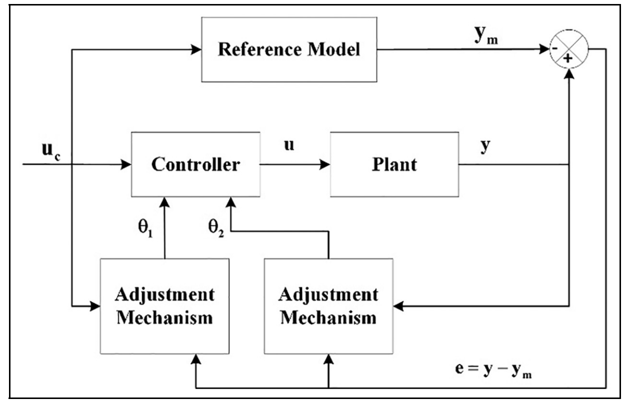

Figure 3. Block diagram model for Lyapunov rule.

Adaptation mechanism

This mechanism finds use in fine-tuning the control algorithm's settings. Adaptation rules obtain the parameters for the same system response as the reference model. It is planned so that the stability of the control system is guaranteed and converges to zero steady-state error. To generate the adaptation mechanism, different types of mathematical methods may be used like augmented error theory (Aydin and Gurleyen, 2018; Mirkin, 2005; Monopoli, 1974), Lyapunov theory (Cui et al., 2022), and MIT rule (Sethi et al., 2017). The adaptation mechanism developed in this research uses both the Lyapunov rule and the MIT rule.

Mathematical modeling

This work uses the MIT, and Lyapunov approaches to build the MRAC scheme for the first- and second-order systems. Theoretically, an under-damped second-order system may be described as oscillatory in its physical behavior. In the event, the oscillations are not reduced throughout the given time. The system may be unstable. Hence, maximum overshoot should be as minimum as possible (ideally zero) for stable operation. It may automatically reduce the system's transient period and improve its performance. Like the characteristics of the first-order system, oscillations are absent in the critically damped second-order system's characteristics. This research uses an under-damped first- and second-order system with high maximum overshoot, considerable settling time, and insufferable dynamic error.

Furthermore, utilizing the MRAC as a mechanism to effect performance improvements throughout the system is the primary focus. A critically damped system has been considered a reference model for comparison since it is appropriate for the objective. Let the equations (1) and (2) given below characterize the first-order and second-order system, respectively

$$\frac{dy_1(t)}{dt} = -a_1 y_1(t) + b_1 u(t) \tag{1}$$

$$\frac{d^2y_2(t)}{dt^2} = -a_2\frac{dy_2(t)}{dt} - b_2 y_2(t) + b_2 u(t) \tag{2}$$

Reference models are given by equations (3) and (4) for the first- and second-order systems, respectively

$$\frac{dy_{1m}(t)}{dt} = -a_{1m} y_{1m}(t) + b_{1m} u_c(t) \tag{3}$$

$$\frac{d^2y_{2m}(t)}{dt^2} = -a_{2m}\frac{dy_{2m}(t)}{dt} - b_{2m} y_{2m}(t) + b_{2m} u_c(t) \tag{4}$$

here, a_1, b_1, a_2, b_2, a_{1m}, b_{1m}, a_{2m}, and b_{2m} are the constants. The equation that describes the control law is shown below by equation (5)

$$u = \theta_1 u_c(t) - \theta_2 y(t) \tag{5}$$

here, $y(t) = y_1(t)$ and $y_m(t) = y_{1m}(t)$ for first-order system and $y(t) = y_2(t)$ and $y_m(t) = y_{2m}(t)$ for the second-order system.

The difference between the output of the reference model, y_m, and the plant output, y, is defined as the term "error function." It may be represented as follows by equation (6)

$$e = y - y_m \tag{6}$$

MIT rule

This rule was devised by the Massachusetts Institute of Technology (MIT); therefore, it is commonly known as the MIT rule. It applies the MRAC scheme (Mukherjee et al., 2018b; Sethi et al., 2017) to real-world systems. For the stability analysis of the system by MIT rule, we needed a loss function J, often known as the cost function, which may be illustrated using (Fan and Kobayashi, 1998; Karthikeyan et al., 2012; Mfoumboulou 2021; Rothe et al., 2020; Zareh and Soheili 2011)

$$J(\theta) = \frac{1}{2}e^2 \tag{7}$$

$$\frac{\partial J}{\partial e} = e \tag{8}$$

where e is the output error, which may be considered as the difference between the output of the plant and the output of the reference model, and θ (i.e. θ_1 and θ_2) is the regulating parameter that is generally recognized as the control parameter. In this case, the loss function is reduced by adjusting a parameter denoted by θ (i.e. θ_1 and θ_2). Therefore, adjusting the parameter so that it moves in the opposite direction as J's gradient would be appropriate, that is

$$\frac{d\theta}{dt} = -\gamma \frac{\partial J(\theta)}{\partial \theta} \tag{9}$$

$$= -\gamma e \frac{\partial e(\theta)}{\partial \theta} \tag{10}$$

where $\partial e/\partial\theta$ is known as the sensitivity derivative of the plant. This term depicts how the error is affected by modification made in the parameter, denoted by the symbol θ. When selecting the loss function J, one has numerous options. As an illustration, it may also play a role in producing errors. Likewise, $d\theta/dt$ may have various relationships according to usage. γ is the adaptation gain (Dinakin and Oluseyi, 2021). Adaptation gain refers to a tuning parameter used to adjust the adaptation rate of the controller. The adaptation gain aims to strike a balance between fast convergence and stability. A high adaptation gain can lead to quick parameter updates and quicker convergence, but it may also introduce instability and overshoot in the control system. On the contrary, a low adaptation gain can improve stability but may result in slower convergence and reduced tracking performance. The selection of the adaptation gain depends on the characteristics of the controlled system and the desired performance specifications. The best adaptation gain can be determined using a systematic strategy, such as trial-and-error, optimization algorithms, or advanced control design methods. In this paper, we have utilized the trial-and-error method to find a suitable range of adaption gain.

Sign-sign algorithm

$$\frac{d\theta}{dt} = -\gamma sign\left(\frac{\partial e}{\partial \theta}\right) sign(e) \tag{11}$$

Alternatively, it could select

$$\frac{d\theta}{dt} = -\gamma \left(\frac{\partial e}{\partial \theta}\right) sign(e) \tag{12}$$

where $sign(e) = -1$; for $e < 0$, $sign(e) = 0$; for $e = 0$, and $sign(e) = 1$; for $e > 0$.

It is possible to get that the selection of adaptation gain is a delicate process and the degree of its dependence on the signal points. Therefore, it could enhance the MIT rule in the following ways

$$\frac{d\theta}{dt} = -\gamma \phi e \tag{13}$$

Here, $\phi = \frac{de}{d\theta}$
Also

$$\frac{d\theta}{dt} = -\frac{\gamma \phi e}{(\beta + \phi^T \phi)} \tag{14}$$

here, $\Phi^T \Phi$ is small, and $\beta > 0$ is taken to evade the zero division.

Since the error is specified by equation (6), hence the modification in error w.r.t. time may be written as

$$\dot{e} = -a_{1m} e(t) - (b_1\theta_2 + a_1 - a_{1m}) y_1(t) + (b_1\theta_1 - b_{1m}) u_c(t) \tag{15}$$

The aim of an adaptive controller is that the y_1 should be the asymptotic trace of y_{1m} and stable, hence from equation (15)

$$b_1\theta_1 - b_{1m} = 0 \Rightarrow \theta_1 = \frac{b_{1m}}{b_1} \tag{16}$$

$$b_1\theta_2 + a_1 - a_{1m} = 0 \Rightarrow \theta_2 = \frac{a_{1m} - a_1}{b_1} \tag{17}$$

Now, $\dot{e} = -a_{1m} e(t)$ is the negative definite and $e \to 0$ as $t \to \infty$, hence system will be stable.

From equation (10)

$$\frac{d\theta_1}{dt} = -\gamma e \frac{\partial e(\theta)}{\partial \theta_1} \tag{18}$$

$$\frac{d\theta_2}{dt} = -\gamma e \frac{\partial e(\theta)}{\partial \theta_2} \tag{19}$$

Putting equation (5) into equation (1) and using the Laplace transform, we get the following

$$y_1(s) = \frac{b_1\theta_1}{s + a_1 + b_1\theta_2} u_c \tag{20}$$

Taking Laplace transform of equation (3)

$$y_{1m}(s) = \frac{b_{1m}}{s + a_{1m}} u_c \tag{21}$$

Taking Laplace transform of equation (6) and putting equations (20) and (21) in equation (6)

$$e(s) = \frac{b_1\theta_1}{s + a_1 + b_1\theta_2} u_c - \frac{b_{1m}}{s + a_{1m}} u_c \tag{22}$$

$$\frac{\partial e}{\partial \theta_1} = \frac{b_1}{s + a_1 + b_1\theta_2} u_c \tag{23}$$

By substituting equation (17) in equation (23)

$$\frac{\partial e}{\partial \theta_1} = \frac{b_1}{s + a_{1m}} u_c \tag{24}$$

Similarly

$$\frac{\partial e}{\partial \theta_2} = -\frac{b_1^2\theta_1}{(s + a_1 + b_1\theta_2)^2} u_c \tag{25}$$

By substituting θ_2 from equation (17) and y_1 from equation (20)

$$\frac{\partial e}{\partial \theta_2} = -\frac{b_1}{s + a_{1m}} y_1 \tag{26}$$

After making the necessary changes to equations (18) and (23) by inserting equations (24) and (26), respectively, (Nguyen, 2018) the new form of the MIT adaption law is as follows

$$\frac{d\theta_1}{dt} = -\gamma e \frac{b_1}{s + a_{1m}} u_c \tag{27}$$

$$\frac{d\theta_2}{dt} = \gamma e \frac{b_1}{s + a_{1m}} y_1 \tag{28}$$

here, b_1 is the system's parameter, which may be absorbed by the equation: $\alpha = \gamma b_1 / a_{1m}$. And hence equations (27) and (28) become equations (29) and (30), respectively

$$\frac{d\theta_1}{dt} = -\alpha e \frac{a_{1m}}{s + a_{1m}} u_c \tag{29}$$

$$\frac{d\theta_2}{dt} = \alpha e \frac{a_{1m}}{s + a_{1m}} y_1 \tag{30}$$

The adaptation law for a second-order system utilizing the MIT rule by following the above approach is described as follows.

By differentiating equation (6) and substituting the values of $\dot{y}$ and $\dot{y}_m$, and rearranging the equation, we get

$$\dot{e} = -\frac{1}{a_2}((\ddot{y}_2 - \ddot{y}_{2m}) - b_{2m}e - (b_2\theta_2 + b_2 - b_{2m})y_2 + (b_2\theta_1 - b_{2m})u_c) \tag{31}$$

The aim of an adaptive controller is that the y_2 should be the asymptotic trace of y_{2m} and hence from equation (30)

$$\theta_1 = \frac{b_{2m}}{b_2} \text{ and } \theta_2 = \frac{b_{2m} - b_2}{b_2} \tag{32}$$

By putting equation (5) in equation (2) and taking Laplace transform

$$y_2(s) = \frac{b_2\theta_1}{s^2 + a_2 s + b_2\theta_2 + b_2} u_c \tag{33}$$

Taking Laplace transform of equation (4)

$$y_{2m}(s) = \frac{b_{2m}}{s^2 + a_{2m}s + b_{2m}} u_c \tag{34}$$

By putting the values of $y_2(s)$ and $y_{2m}(s)$ in Laplace transform of equation (6)

$$e(s) = \frac{b_2\theta_1}{s^2 + a_2 s + b_2\theta_2 + b_2} u_c - \frac{b_{2m}}{s^2 + a_{2m}s + b_{2m}} u_c$$

$$\frac{de}{d\theta_1} = \frac{b_2}{s^2 + a_2 s + b_2\theta_2 + b_2} u_c \tag{35}$$

By substituting the value of θ_2

$$\frac{de}{d\theta_1} = \frac{b_2}{s^2 + a_2 s + b_{2m}} u_c \tag{36}$$

$$\frac{de}{d\theta_2} = -\frac{b_2^2\theta_1}{(s^2 + a_2 s + b_2\theta_2 + b_2)^2} u_c$$

$$\frac{de}{d\theta_2} = \left(\frac{b_2\theta_1}{s^2 + a_2 s + b_2\theta_2 + b_2} u_c\right)\left(-\frac{b_2}{s^2 + a_2 s + b_2\theta_2 + b_2}\right)$$

$$\frac{de}{d\theta_2} = -\left(\frac{b_2}{s^2 + a_2 s + b_2\theta_2 + b_2} y_2\right) \tag{37}$$

Now substituting $\frac{de}{d\theta_1}$ and $\frac{de}{d\theta_2}$ in equations (18) and (19), respectively

$$\frac{d\theta_1}{dt} = -\alpha e \frac{b_{2m}}{s^2 + a_2 s + b_{2m}} u_c \tag{38}$$

$$\frac{d\theta_2}{dt} = \alpha e \frac{b_{2m}}{s^2 + a_2 s + b_{2m}} y_2 \tag{39}$$

where $\alpha = \gamma b_2 / b_{2m}$. Equations (38) and (39) may also be written as

$$\frac{d\theta_1}{dt} = -\alpha e \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2} u_c$$

$$\frac{d\theta_2}{dt} = \alpha e \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2} y_2$$

where ω_n is the natural frequency, ζ is the damping ratio, and $b_{2m} = \omega_n^2$ and $a_2 = 2\zeta\omega_n$.

A filter, which represents the transfer function of the first-order and second-order reference model, is adopted by the adaptation law for first-order and second-order systems that use the MIT rule. Figure 2 shows the block diagram of MRAC using MIT rule.

Lyapunov rule

The Lyapunov stability theory (Ma et al., 2022) may be used to describe the algorithms for adjusting parameters in the MRAC system. For the systems mentioned above (equations (1) and (2)), the controller law is defined by equation (5), and the error is given by equation (6).

Now, we may write the change in error w.r.t. time as

$$\dot{e} = -a_m e - (b\theta_2 + a - a_m)y + (b\theta_1 - b_m)u_c \tag{40}$$

The Lyapunov function is described by

$$V(e, \theta_1, \theta_2) = \frac{1}{2}\left(e^2 + \frac{1}{b\gamma}(b\theta_2 + a - a_m)^2 + \frac{1}{b\gamma}(b\theta_1 - b_m)^2\right) \tag{41}$$

The Lyapunov function V to be positive-definite b should be positive (i.e. $b > 0$).

where

$$e = y - y_m \text{ and } \gamma = \text{Adaptation gain of the controller}$$

$$\theta_1 = \frac{b_m}{b} \text{ and } \theta_2 = \frac{a_m - a}{b}$$

$$a = \begin{cases} a_1, & (for\ first - order\ system) \\ a_2, & (for\ second - order\ system) \end{cases} and$$

$$a_m = \begin{cases} a_{1m}, & (for\ first - order\ system) \\ a_{2m}, & (for\ second - order\ system) \end{cases}$$

$$b = \begin{cases} b_1, & (for\ first - order\ system) \\ b_2, & (for\ second - order\ system) \end{cases} and$$

$$b_m = \begin{cases} b_{1m}, & (for\ first - order\ system) \\ b_{2m}, & (for\ second - order\ system) \end{cases}$$

By differentiating equation (41) and subsuming equation (40)

$$\dot{V} = -a_m e^2 + \frac{1}{\gamma}(b\theta_2 + a - a_m)(\dot{\theta}_2 - \gamma y e) + \frac{1}{\gamma}(b\theta_1 - b_m)(\dot{\theta}_1 + \gamma u_c e) \tag{42}$$

To ensure that $\dot{V}$ is negative definite

$$\frac{1}{\gamma}(b_m\theta_2 + a + a_m)(\dot{\theta}_2 - \gamma y e) + \frac{1}{\gamma}(b\theta_1 - b_m)(\dot{\theta}_1 + \gamma u_c e) = 0 \tag{43}$$

Therefore, equations (44) and (45) define adaptation laws using the Lyapunov rule, where $\alpha = \gamma b/a_m$ for both first-order system and second-order system. Figure 3 shows the block diagram of MRAC using Lyapunov rule.

$$\frac{d\theta_1}{dt} = -\alpha e u_c \tag{44}$$

$$\frac{d\theta_2}{dt} = -\alpha e y \tag{45}$$

Performance evaluation and simulation results

In this paper, the MRAC model has been simulated in MATLAB and Simulink. The MIT and Lyapunov rules are applied separately to the first- and second-order systems. This section details an experimental performance evaluation of the adaptive controller. It investigated how different adaptation gain values affect the system behavior, both for the MIT rule and the Lyapunov adaptation strategy. Second, changes in the desired system response as defined by the reference model's parameters a_m and b_m have also been investigated.

The adaptive controllers are analyzed using a step input signal by taking a first-order system as $a_1 = 3$ and $b_1 = 2$ (Swarnkar et al., 2010) and a second-order system as $a_2 = 8$ and $b_2 = 600$ (Swarnkar et al., 2011) which is represented by equations (46) and (47), respectively

$$y_1(s) = \frac{2}{s+3} \tag{46}$$

$$y_2(s) = \frac{600}{s^2 + 8s + 600} \tag{47}$$

In MRAC, the first step is to select the reference model depending on the requirement. After that, the control algorithm's design is done to update the controller's adjustable parameters. The reference model regarding the transfer function obtained from the desired performance specifications (i.e. rise time, settling time, overshoot, and steady-state error) is given. In the present work, for the analysis, the reference model of the first-order and second-order system is taken from Dinakin and Oluseyi (2021) and Simon (2010), respectively.

Influence of adaptation gain using MIT rule and Lyapunov rule on first-order system

At first, the adaptation gain α is varied, and the resulting influence on the first-order system's (46) time response is analyzed. The results of an experiment with gain values 0.1, 1, 5, 10, 20, 50, 100, 200, 500, 1000, 10,000, and 11,000 have been shown in Figure 4 with MIT rule and in Figure 6 with the Lyapunov method for the reference model $a_m = 4$ and $b_m = 4$ (Swarnkar et al., 2010).

After thorough observations from Figures 4–11 and Table 1, the following inference has been made:

- Figure 4 shows the step response of the first-order system (46) for the range of adaptation gain from 0.1 to 11,000 with the MIT rule. It may be observed that as the adaptation gain increases, the settling time and overshoot decrease.
- At the same time, oscillation and overshoot occurred for higher adaptation gain values which are seen by the output error signal as depicted in Figure 5.
- Figures 8 and 9 show the comparative representation of the effect of adaptation gain with the MIT rule.
- Figure 6 shows the step response of the first-order system (46) for the range of adaptation gain from 0.1 to 11,000 with the Lyapunov rule. It may be observed that as the adaptation gain increases, the settling time and overshoot decrease sharply compared to the MIT rule.
- At the same time, oscillation and overshoot occurred for higher adaptation gain values but less as compared to the MIT rule, which is seen by the output error signal, as shown in Figure 7.
- Figures 10 and 11 show the comparative representation of the effect of adaptation gain with the Lyapunov rule.
- From the above observations, it has been concluded that the response obtained with the Lyapunov rule is far superior compared to the MIT rule in terms of overshoot, peak time, rise time, and settling time.

Influence of changing reference model parameters using MIT and Lyapunov on first-order system

After analyzing the effect of adaptation gain on the first-order system, some critical adaption gain is selected for further investigation based on the change in the reference model. The influence of changing the reference model's parameters

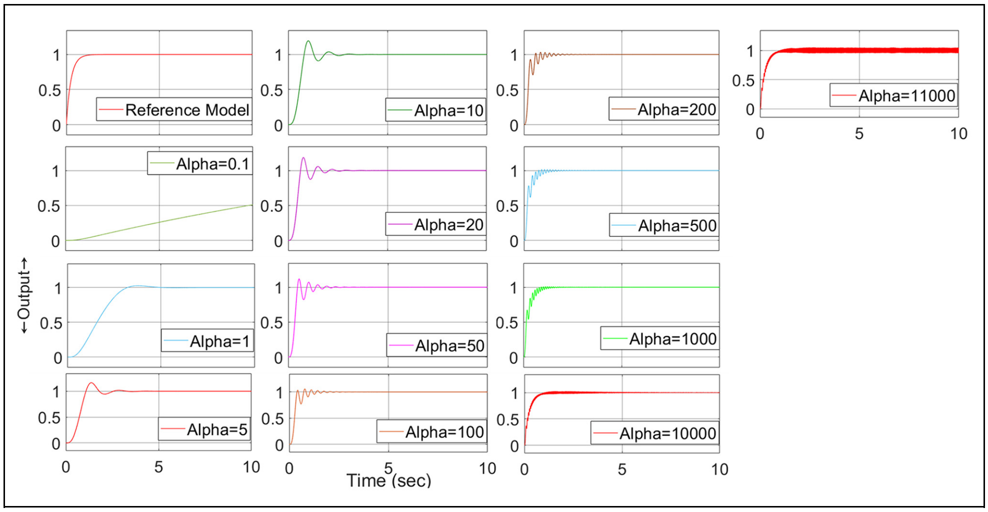

Figure 4. Simulation results of MRAC with MIT rule for various values of adaptation gain (α).

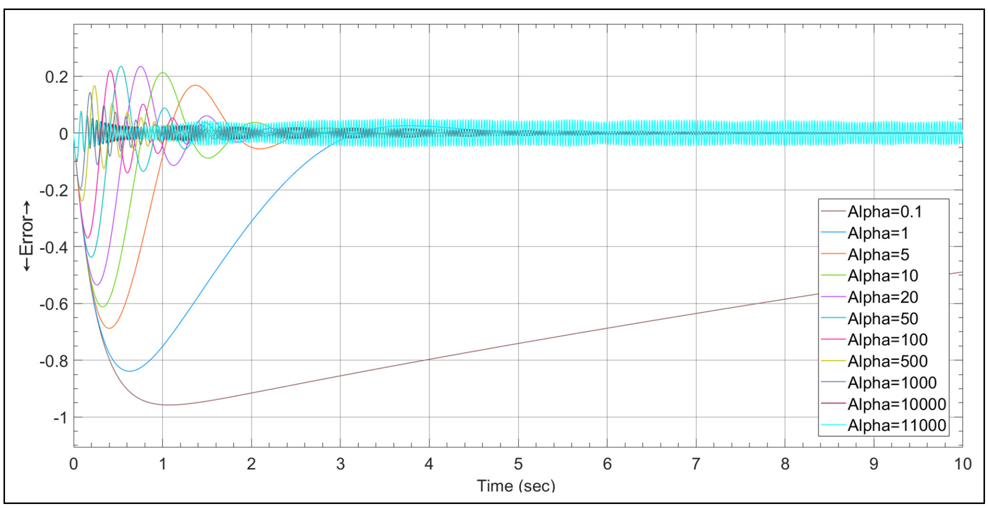

Figure 5. Output error of MRAC with MIT rule for various values of adaptation gain (α).

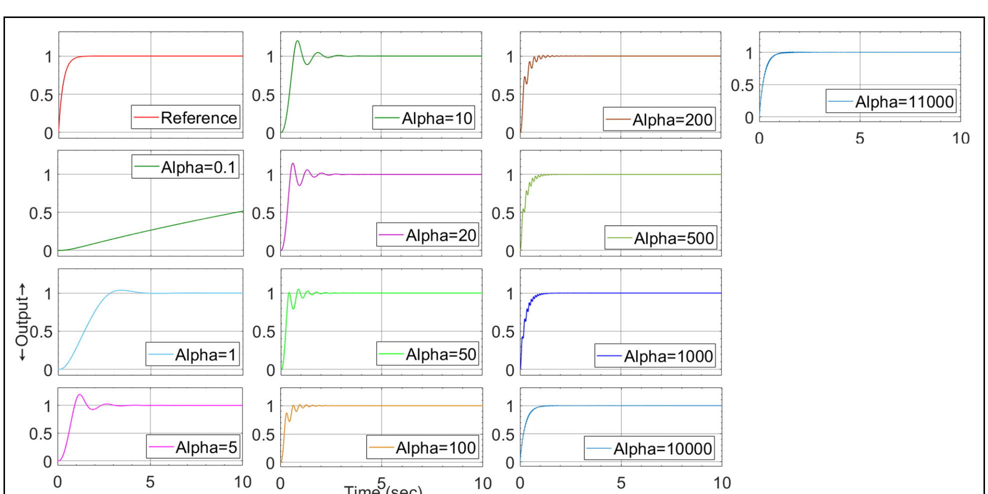

Figure 6. Simulation results of MRAC with Lyapunov rule for various values of adaptation gain (α).

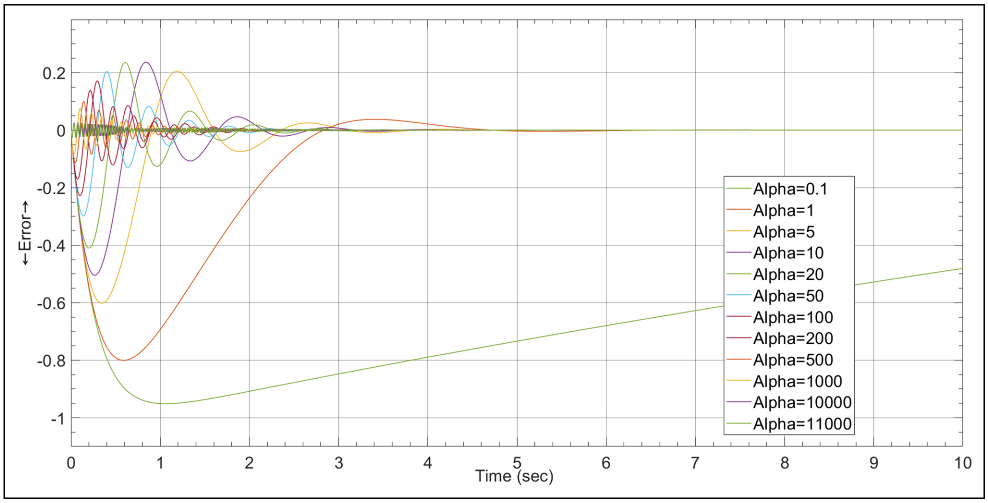

Figure 7. Output error of MRAC with Lyapunov rule for various values of adaptation gain (α).

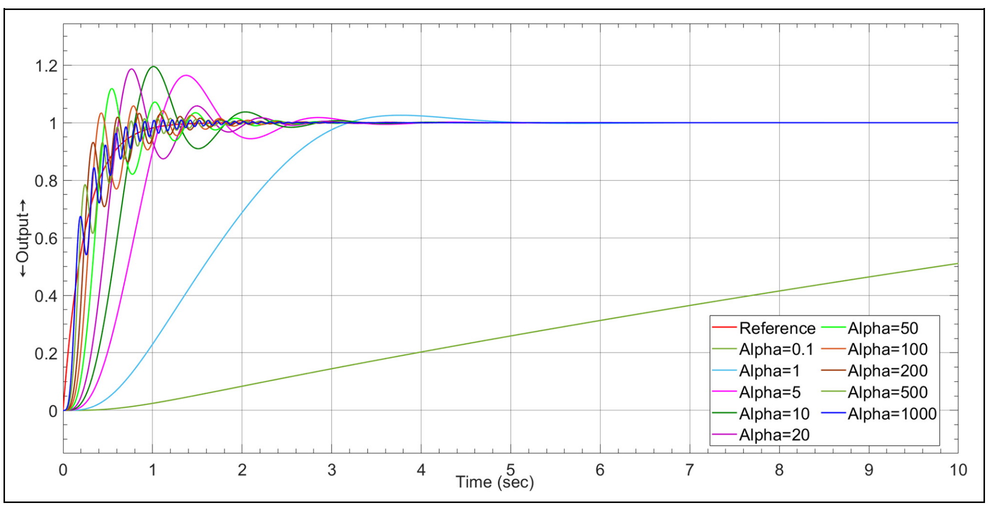

Figure 8. Simulation results of MRAC with MIT rule for various values of adaptation gain (α).

Table 1. Effect of adaptation gain on first-order system with MIT rule and Lyapunov rule.

Adaptation law	MIT rule				Lyapunov rule			
Adaptation gain	Overshoot	Peak time	Settling time	Rise time	Overshoot	Peak time	Settling time	Rise time
0.1	0	10	9.7801	7.4521	0	10	9.7773	7.5481
1	2.5761	3.7692	4.1612	1.9447	3.7680	3.4084	4.0360	1.8056
5	16.4387	1.3732	2.4198	0.6140	19.6282	1.19445	2.8109	0.5593
10	19.5190	1.0088	2.2097	0.4359	20.2399	0.8536	2.4142	0.3945
20	18.6801	0.7662	1.9600	0.3272	14.9875	0.6241	1.8064	0.2953
50	11.8677	0.5433	1.7508	0.2392	5.2183	0.8777	1.5874	0.2215
100	5.8598	0.7856	1.6160	0.1982	2.2578	0.9661	1.1834	0.4653
200	3.2669	0.8502	1.4432	0.1772	0.8862	1.1572	1.2777	0.3683
500	1.6151	1.0554	1.2876	0.3158	0.2611	1.4729	1.1344	0.5231
1000	1.0081	1.2190	1.2776	0.3730	0.1112	1.6668	1.0377	0.4968
10,000	2.6092	1.5614	3.0086	0.4630	0.4221	1.5639	1.0459	0.5242
11,000	5.3758	2.7298	9.9978	0.4792	0.7134	1.5833	1.0882	0.5292

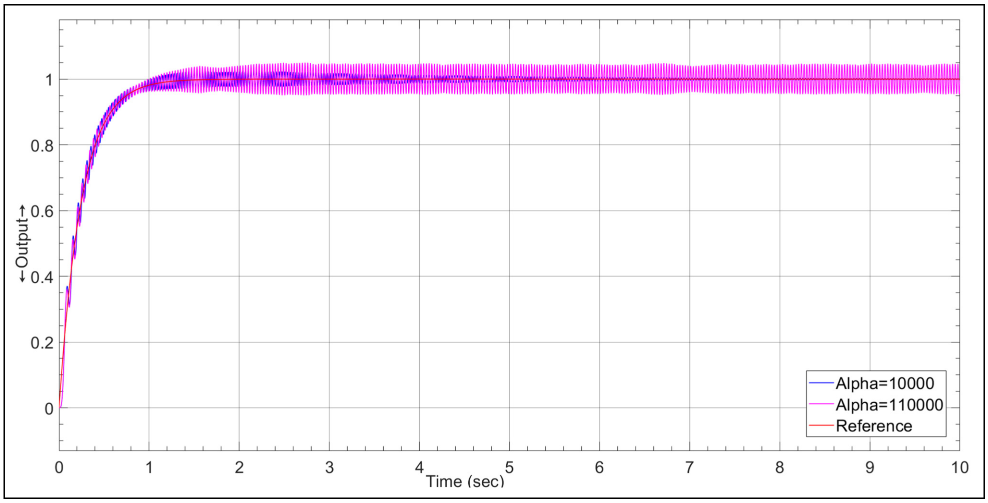

Figure 9. Simulation results of MRAC with MIT rule for 10,000 and 11,000 values of adaptation gain (α).

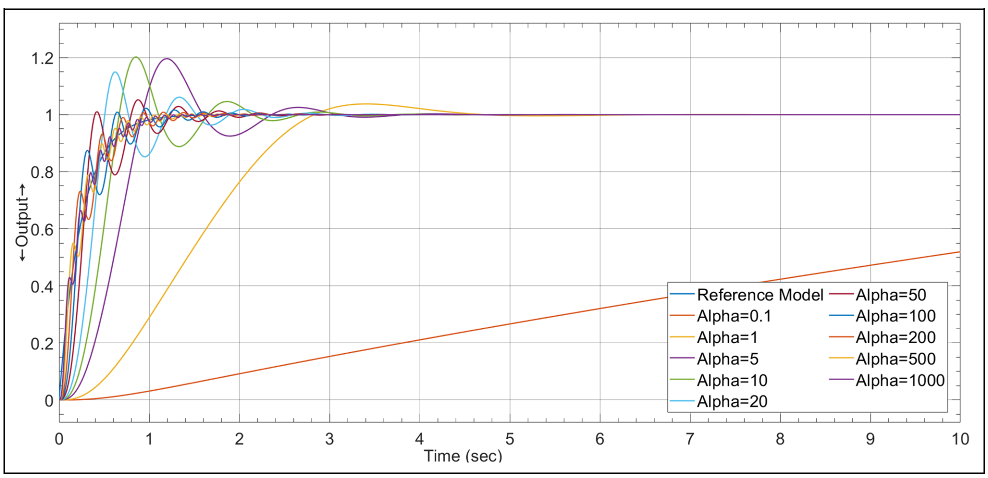

Figure 10. Simulation results of MRAC with Lyapunov rule for various values of adaptation gain (α).

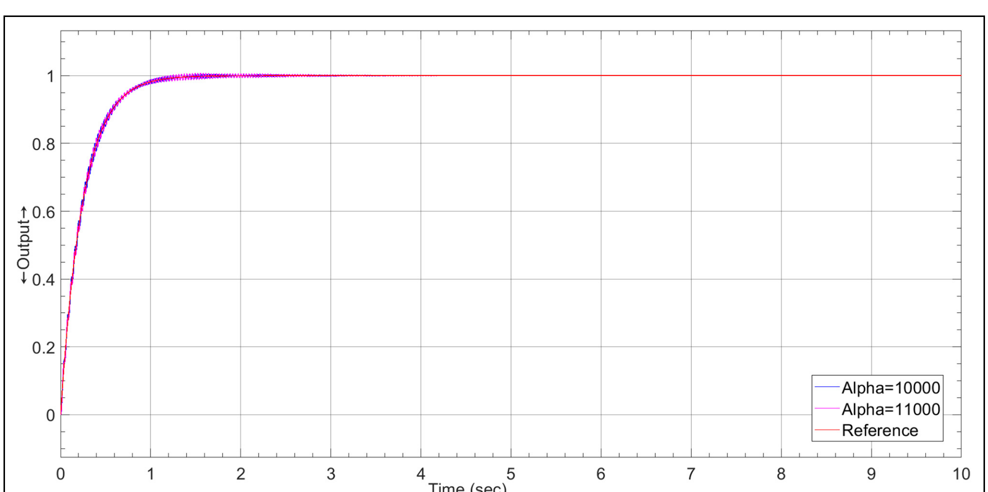

Figure 11. Simulation results of MRAC with Lyapunov rule for 10,000 and 11,000 values of adaptation gain (α).

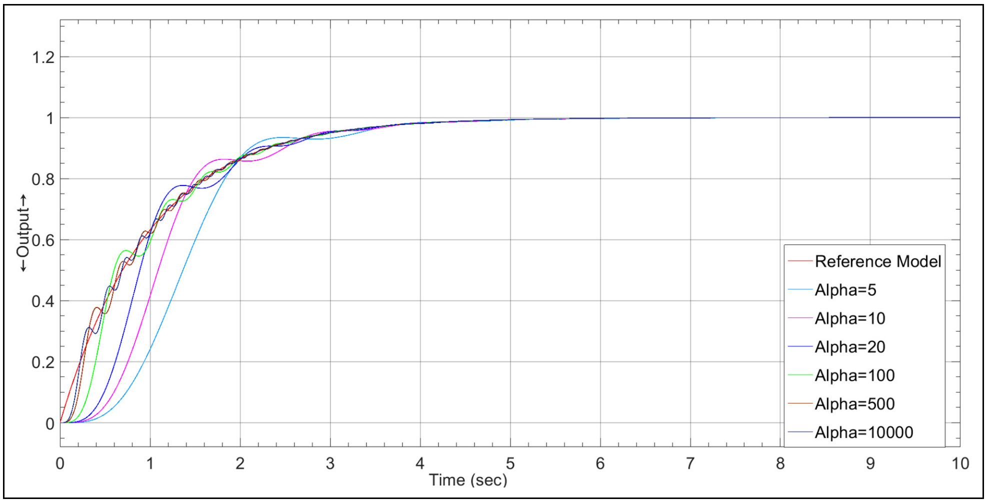

Figure 12. Simulation results of MRAC with MIT rule for various values of adaptation gain (α) for reference model $a_m = b_m = 1$.

a_m and b_m has also been examined. As before, the plant process parameters are set to $a = 3$, $b = 2$, and the adaptation gain is now chosen as $\alpha = 5$, 10,100, 500, 1000, and 10,000.

From Figures 12–21 and Table 2, the following conclusions have been drawn:

- Figures 12–16 show the step response of the first-order system with MIT rule, and from Figures 17–21 show the step response of the first-order system with Lyapunov rule for the reference model $a_m = b_m = 1$, $a_m = b_m = 4$, $a_m = b_m = 10$, and $a_m = b_m = 100$, respectively, for the $\alpha = 5$, 10, 100, 500, 1000, and 10,000.
- From the above results, it may be observed that for the lower value of the time constant of the reference model, system response has been tracked well to the reference model for the selected range of adaptation gain.
- As the reference model's time constant increases, the overshoot also increases, but the system's response approaches the steady-state value faster for a higher adaptation gain.

Influence of changing reference model parameters using MIT and Lyapunov on second-order system

Here, the effect of changing the reference model's parameters a_m and b_m is examined. The plant process parameters are set to $a = 8$ and $b = 600$ (Swarnkar et al., 2011), and the adaptation gain is now chosen as $\alpha = 1, 2, 3, 4$, and 5.

The step response of the second-order system (47) for the four deferent references ($a_m = 1, 2, 3$, and 4 and $b_m = 1, 4, 9$, and 16, respectively) and adaptation gain 1, 2, 3, 4 and 5 are shown from Figures 22–25 using the MIT rule and from Figures 26–29 using the Lyapunov rule. From these results, the following observations may be drawn:

- Figures 22 and 23 show the system's step response (47) for the reference $a_m = 2$ and $b_m = 1$ for a range of adaptation gain from 1 to 5 and 1 to 4, respectively, using the MIT rule. Here, it may be observed that the system's response is a stable and asymptotic trace of the reference model for the range of adaptation gain from 1 to 3, and the system became unstable at $\alpha = 4$ and 5.
- Figures 24 and 25 show the system's step response (47) for the reference $a_m = 4$ and $b_m = 4$, the range of adaptation gain from 1 to 5 and from 1 to 4, respectively, using the MIT rule. This result shows that the system's response is a stable and asymptotic trace of the reference model for the adaptation gain from 1 to 3, and the system became unstable at $\alpha = 4$ and 5.
- Figure 26 shows the system's step response (47) for the reference model $a_m = 6$ and $b_m = 9$ for the range of adaptation gain from 1 to 5 using the MIT rule. This result shows that the system's response is a stable and asymptotic trace of the reference model for the range of adaptation gain from 1 to 3. Still, the system became unstable at $\alpha = 4$, and $\alpha = 5$ system is a stable and asymptotic trace of reference model with 5% tolerance.
- Figures 27 and 28 show the system's step response (47) for the reference $a_m = 8$ and $b_m = 16$ for the range of adaptation gain from 1 to 5 and from 1 to 4, respectively, using the MIT rule. This result shows that the system's response is a stable and asymptotic trace of the reference model for the adaptation gain from 1 to 3, and the system became unstable at $\alpha = 4$ and 5.
- Figures 29–31 show the step response of system (47) for the reference model $a_m = 2$ and $b_m = 1$; $a_m = 4$; and $b_m = 4$ and $a_m = 6$ and $b_m = 9$, respectively, for

Table 2. Effect of change of parameters of reference model on first-order system with MIT and Lyapunov rules.

Adaptation law		MIT rule				Lyapunov rule			
Reference model	Adaptation gain	Overshoot	Peak time	Settling time	Rise time	Overshoot	Peak time	Settling time	Rise time
$\frac{1}{s+1}$	5	0	10	3.8178	1.3725	0	10	4.0998	2.0845
	10	0	10	3.8710	1.8908	0	10	3.9916	1.8710
	100	0	10	3.9425	1.9915	0	10	3.9290	2.1672
	500	0	10	3.8959	2.1194	0	10	3.9068	2.1863
	1000	0	10	3.9141	2.1322	0	10	3.9149	2.2117
	10,000	0.0735	9.8213	3.9221	2.1935	0.3042	9.8275	3.8976	2.1823
$\frac{4}{s+4}$	5	16.4388	1.3740	2.4198	0.6140	19.6281	1.1966	2.8109	0.5593
	10	19.5189	1.0087	2.2097	0.4359	20.2412	0.8514	2.4142	0.3944
	100	5.5869	0.7846	1.6160	0.1982	2.2586	0.9656	1.1834	0.4652
	500	1.6155	1.0546	1.2876	0.3157	0.2612	1.4727	1.1345	0.5231
	1000	1.0096	1.2200	1.2776	0.3729	0.1117	1.6681	1.0377	0.4968
	10,000	2.9042	1.5026	3.1361	0.4644	0.5234	1.4765	1.0842	0.5293
$\frac{10}{s+10}$	5	22.3546	1.1113	2.6894	0.4948	25.7923	1.0295	2.6866	0.4692
	10	30.2657	0.7846	2.0033	0.3303	34.1155	0.7127	2.3109	0.3105
	100	32.9166	0.3101	0.7738	1.2923	27.3103	0.2526	1.0826	0.1118
	500	13.7330	0.3340	0.9863	0.0762	6.8880	0.2801	0.7102	0.0737
	1000	9.3306	0.2599	0.9237	0.0659	3.2652	0.4100	0.5729	0.1537
	10,000	3.1477	0.5314	1.4748	0.1624	0.4168	0.5968	0.4585	0.1950
$\frac{50}{s+50}$	5	22.7531	0.9884	2.5457	0.4633	25.7923	1.0295	2.6866	0.4692
	10	32.1344	0.6638	1.8825	0.2930	34.1155	0.7127	2.3109	0.3105
	100	55.7380	0.2213	1.0525	0.0832	27.3103	0.2526	1.0826	0.1118
	500	60.9021	0.1153	0.7725	0.0412	6.8880	0.2801	0.7102	0.0737
	1000	58.0956	0.0884	0.7092	0.0317	3.2652	0.4100	0.5729	0.1537
	10,000	28.8239	0.0797	0.6887	0.0156	0.4168	0.5968	0.4585	0.1950
$\frac{100}{s+100}$	5	25.9475	0.9432	2.5876	0.4444	26.4742	0.9339	2.5879	0.4415
	10	35.9378	0.6270	1.8564	0.2800	36.7563	0.6169	2.1791	0.2777
	100	56.1026	0.1903	0.8598	0.0763	58.1064	0.1829	0.8574	0.0748
	500	60.5646	0.0921	0.4009	0.0352	62.7349	0.0849	0.3936	0.0339
	1000	60.2911	0.0694	0.3287	0.0262	61.6658	0.0625	0.2820	0.0248
	10,000	43.7446	0.0295	0.1747	0.0113	34.6103	0.0247	0.1231	0.0103

the range of adaptation gain from 1 to 5 using the Lyapunov rule. This result shows that the system's response is a stable and asymptotic trace of the reference model for the range of $\alpha = 1$ to $\alpha = 5$, but at $\alpha = 4$ and 5, the system oscillates for some time.

- Figure 32 shows the system's step response (47) for the reference $a_m = 8$ and $b_m = 16$ for the range of α from 1 to 5 through the Lyapunov rule. These results show the system is stable only for $\alpha = 1$ to 4. At $\alpha = 4$, the system oscillates to converge to a steady value, but at $\alpha = 5$, the system becomes unstable.

From the above observations, it may be concluded that for the second-order system, the range of adaptation gain will decrease as ω_n of the reference model increases. For the second-order system, the best suitable range of the adaptation gain is from 1 to 3 for MIT and Lyapunov, both separately, as shown in Figures 31 and 32, respectively. For the Lyapunov rule, the second-order system is stable for $\alpha = 4$ and 5, with some oscillation.

Hence, it may be concluded that the second-order system worked very well for the range of adaptation gain of 1 to 3 for all critically damped second-order systems for both the adaptation law MIT and Lyapunov as shown in Figure 33(a) and (b), respectively. From these results, it may be seen that there is no overshoot present in the response, and as the natural frequency of the reference model is increased, the system response is faster. Figure 34(a) and (b) show the output error of system response at $\alpha = 3$ for the reference model of natural frequency from 1 to 5 with the MIT and Lyapunov adaptation algorithms, respectively.

From Table 3, we may observe that the system's response is speedy for the lower values of adaptation gain and becomes slower with some overshoot as the adaptation gain increases. But it may also be observed that if the natural frequency of the reference model is increasing, keeping the damping ratio equal to 1, the system's response becomes very fast, and overshoot is also reduced.

Stability analysis

The step response of the reference and proposed models with and without a controller is shown below from Figure 35(a)–(d).

- From Figures 35(a)–(d), it may be observed that the response of the systems with the controller became

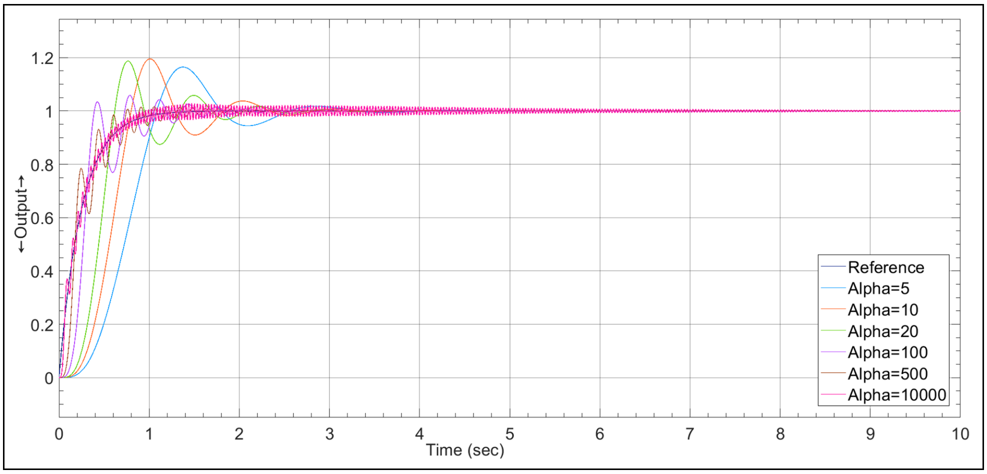

Figure 13. Simulation results of MRAC with MIT rule for various values of adaptation gain (α) for reference model $a_m = b_m = 4$.

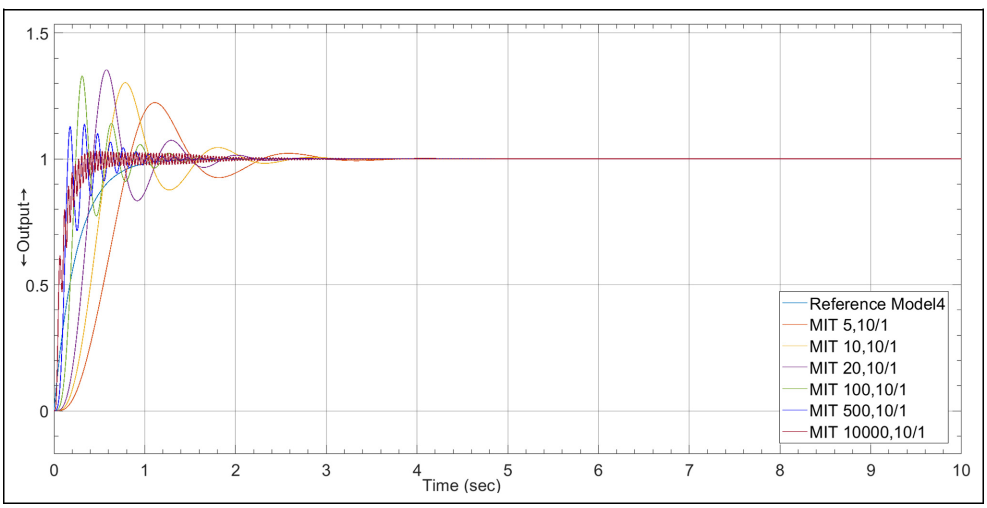

Figure 14. Simulation results of MRAC with MIT rule for various values of adaptation gain (α) for reference model $a_m = b_m = 10$.

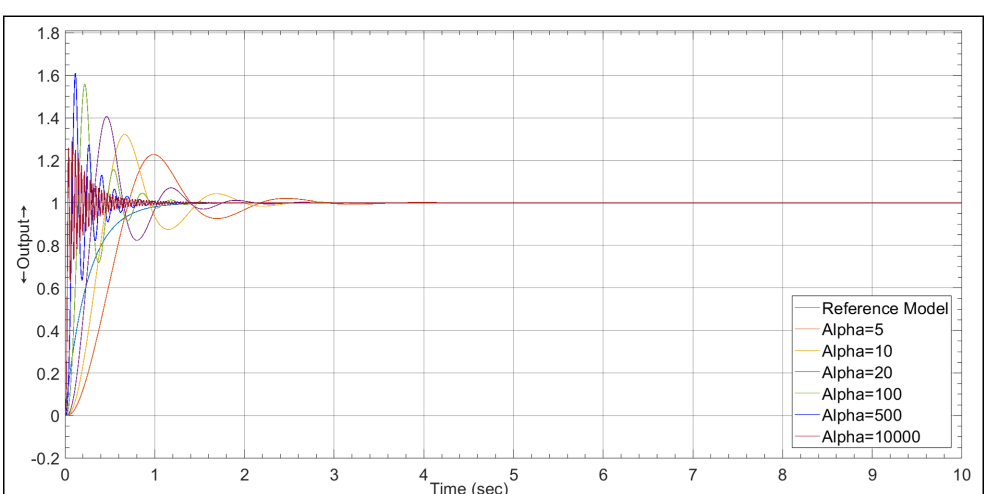

Figure 15. Simulation results of MRAC with MIT rule for various values of adaptation gain (α) for reference model $a_m = b_m = 50$.

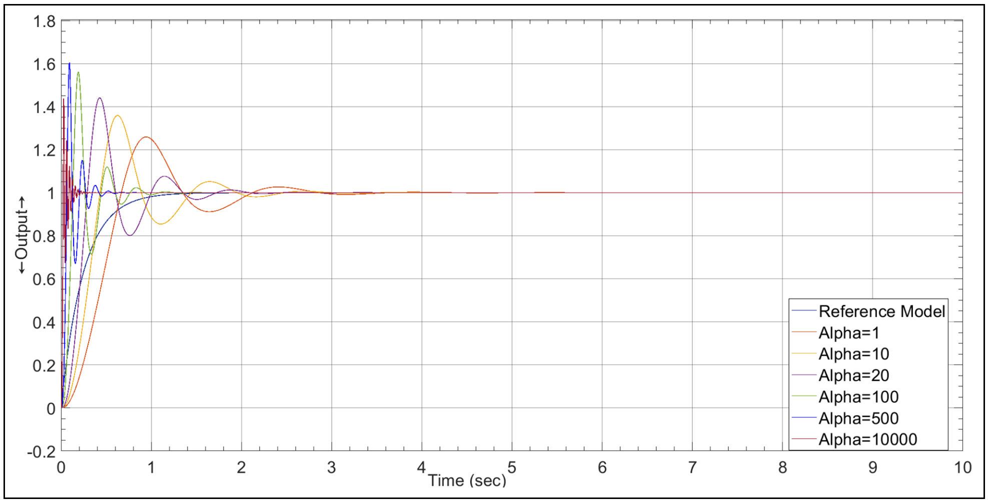

Figure 16. Simulation results of MRAC with MIT rule for various values of adaptation gain (α) for reference model $a_m = b_m = 100$.

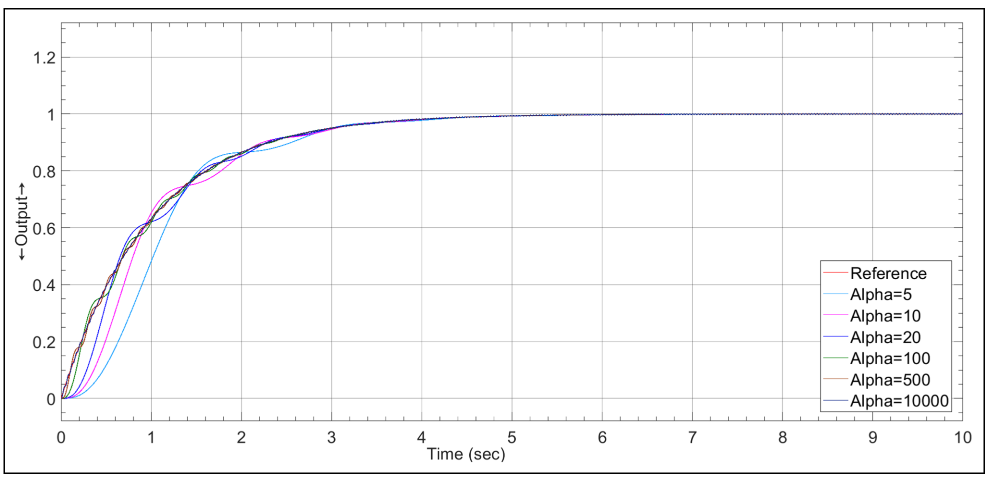

Figure 17. Simulation results of MRAC with Lyapunov rule for various values of adaptation gain (α) for reference model $a_m = b_m = 1$.

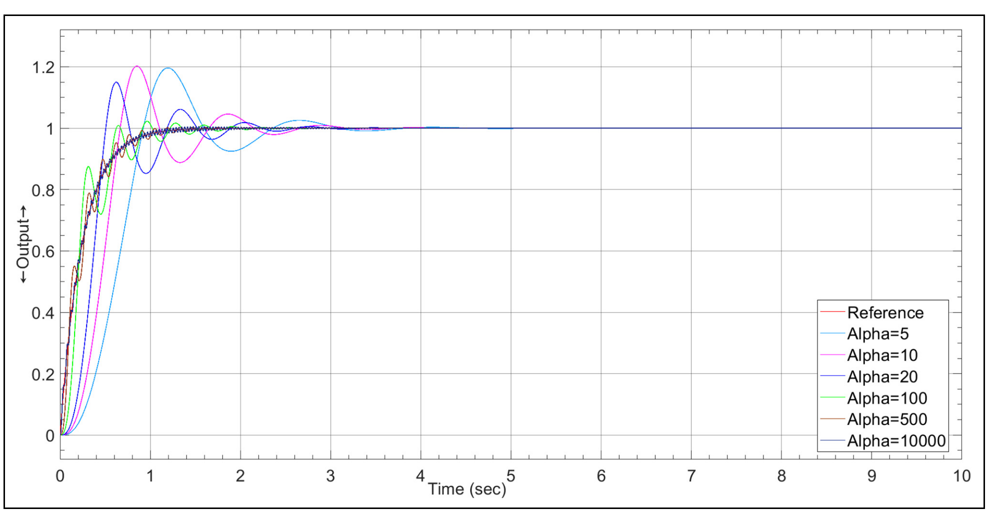

Figure 18. Simulation results of MRAC with Lyapunov rule for various values of adaptation gain (α) for reference model $a_m = b_m = 4$.

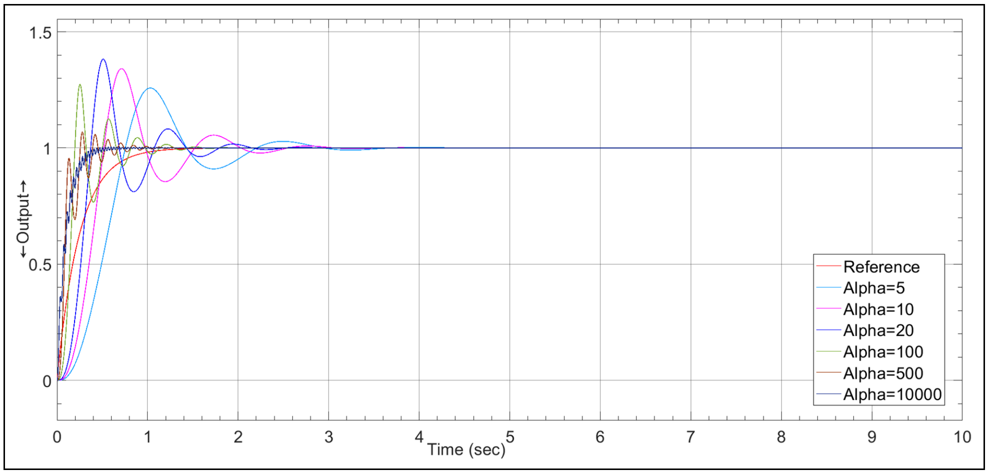

Figure 19. Simulation results of MRAC with Lyapunov rule for various values of adaptation gain (α) for reference model $a_m = b_m = 10$.

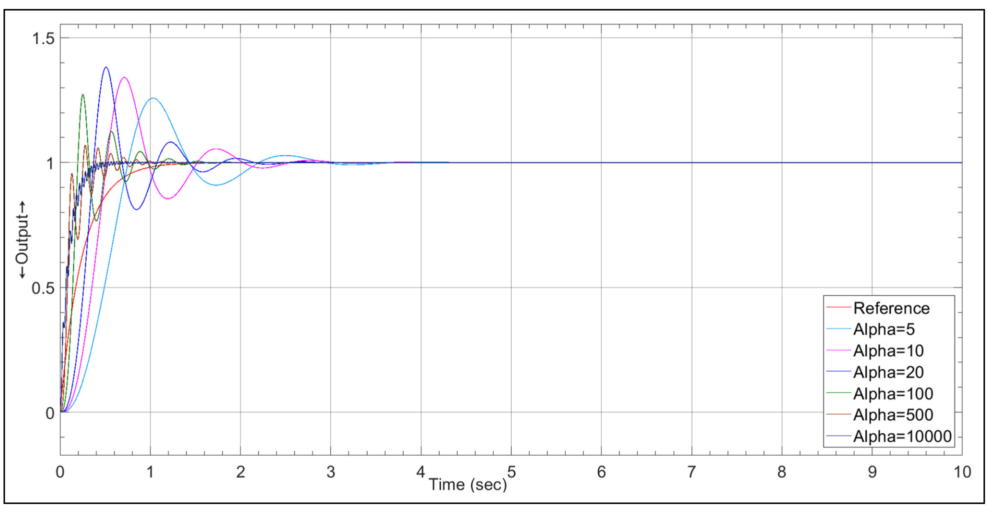

Figure 20. Simulation results of MRAC with Lyapunov rule for multiple values of adaptation gain (α) for reference model $a_m = b_m = 50$.

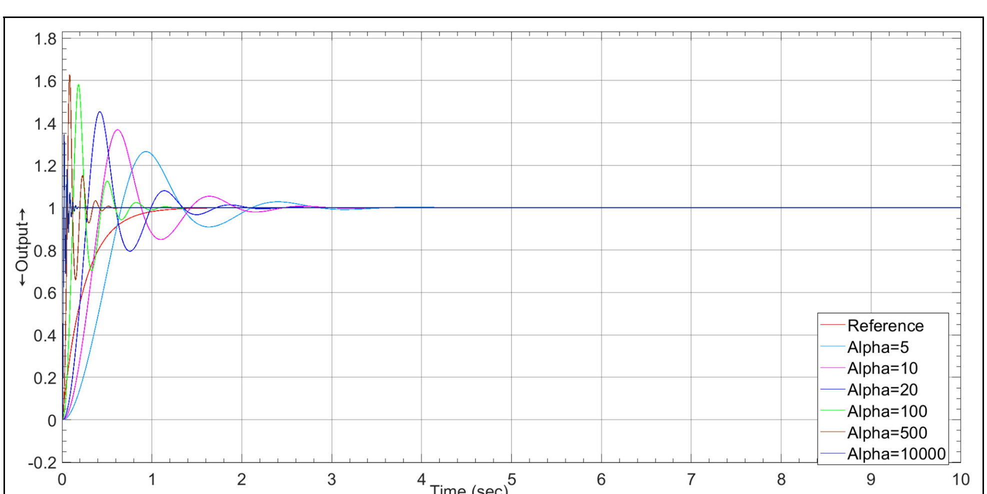

Figure 21. Simulation results of MRAC with Lyapunov rule for various values of adaptation gain (α) for reference model $a_m = b_m = 100$.

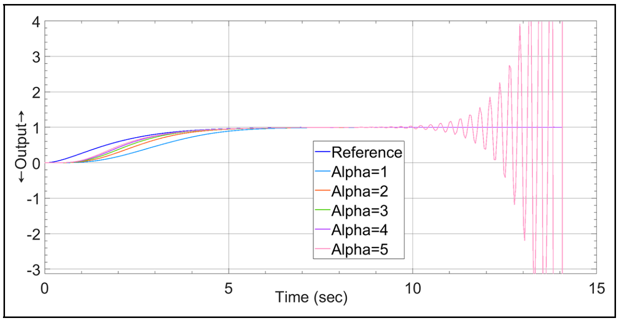

Figure 22. Simulation results of MRAC with MIT rule for reference model $a_m = 2$ and $b_m = 1$ at adaptation gain 1, 2, 3, 4, and 5.

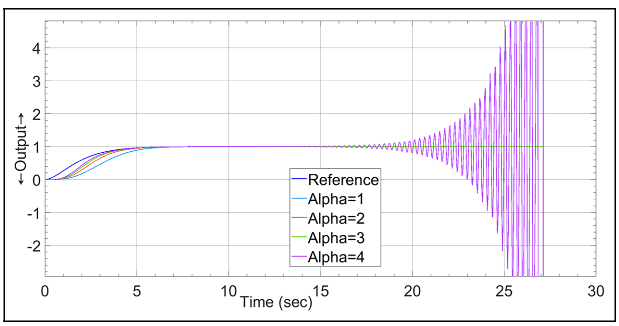

Figure 23. Simulation results of MRAC with MIT rule for reference model $a_m = 2$ and $b_m = 1$ at adaptation gain 1, 2, 3, and 4.

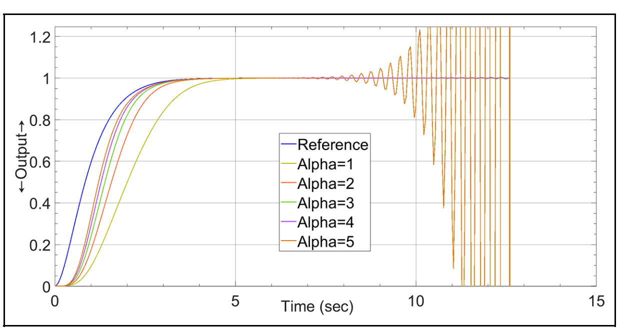

Figure 24. Simulation results of MRAC with MIT rule for reference model $a_m = 4$ and $b_m = 4$ at adaptation gain 1, 2, 3, 4, and 5.

fast, reached the desired goal, and is evidence of the system's stability.

Conclusion

The present work investigates the effect of varying adaptation gains and reference models for the first-order and second-order systems. It has been observed that with the increase in the values of adaptation gain, the system's

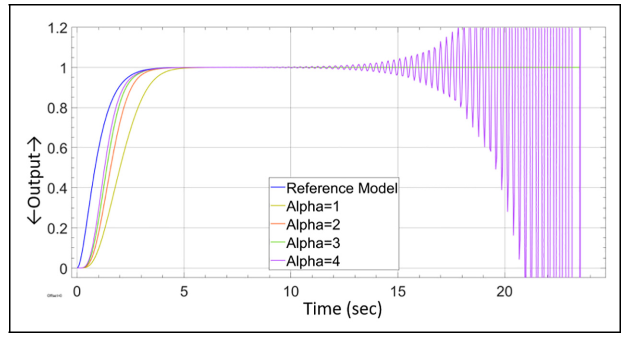

Figure 25. Simulation results of MRAC with MIT rule for reference model $a_m = 4$ and $b_m = 4$ at adaptation gain 1, 2, 3, and 4.

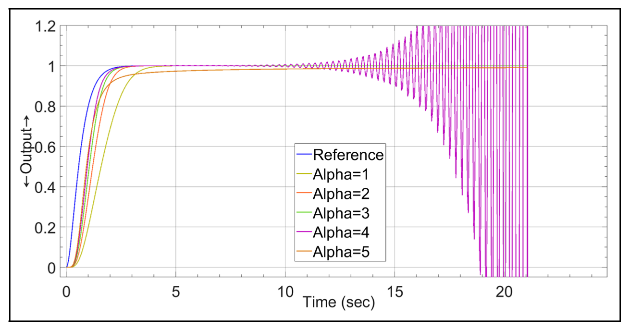

Figure 26. Simulation results of MRAC with MIT rule for reference model $a_m = 6$ and $b_m = 9$ at adaptation gain 1, 2, 3, 4, and 5.

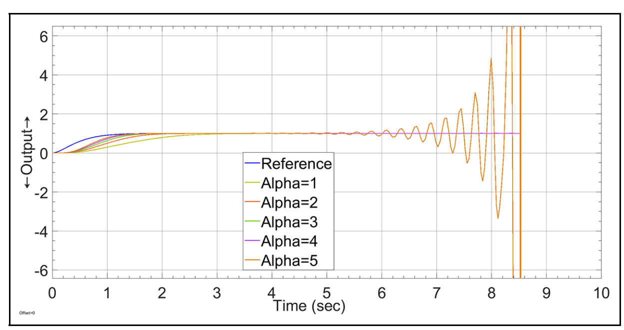

Figure 27. Simulation results of MRAC with MIT rule for reference model $a_m = 8$ and $b_m = 16$ at adaptation gain 1, 2, 3, 4, and 5.

performance is improving in terms of fast response, lesser settling time, and a drop in overshoot. It has also been observed that there are oscillations in the response after a specific gain value. All the investigations have been done using MATLAB/Simulink. After incorporating the variations of the adaptation gain and reference model for the first-order and second-order systems with the MIT rule and Lyapunov rule separately, it is worth mentioning the following salient conclusions:

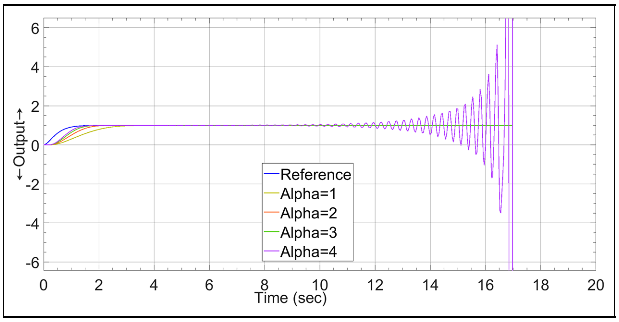

Figure 28. Simulation results of MRAC with MIT rule for reference model $a_m = 8$ and $b_m = 16$ at adaptation gain 1, 2, 3, and 4.

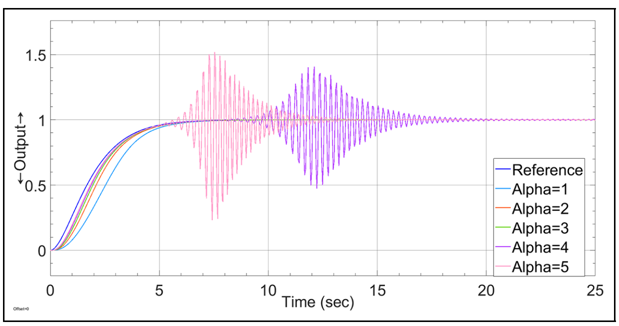

Figure 29. Simulation results of MRAC with Lyapunov rule for reference model $a_m = 2$ and $b_m = 1$ at adaptation gain 1, 2, 3, 4, and 5.

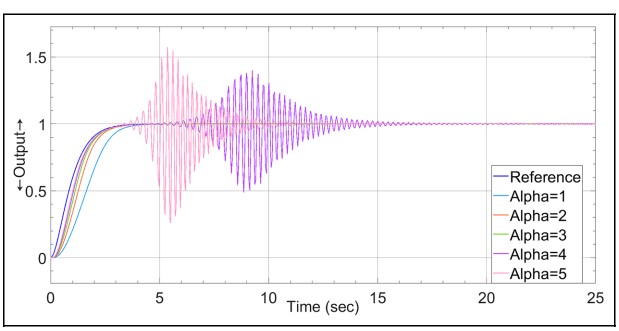

Figure 30. Simulation results of MRAC with Lyapunov rule for reference model $a_m = 4$ and $b_m = 4$ at adaptation gain 1, 2, 3, 4, and 5.

For first-order systems

- First, the investigations are done for adaptation gain by varying the value of α from 1 to 11,000 for a reference model $a_m = b_m = 4$. From the observation of results depicted in Figures 4–11 and Table 1, it may be observed that as the adaptation gain increases, the settling time and overshoot are reduced.

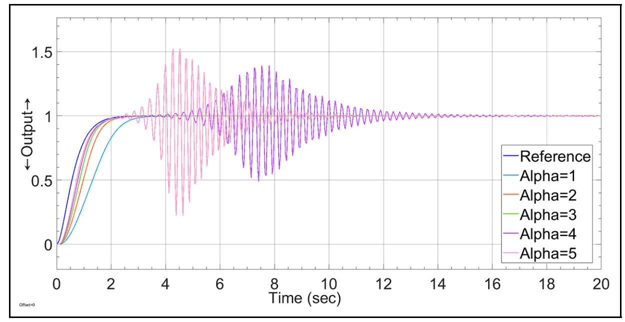

Figure 31. Simulation results of MRAC with Lyapunov rule for reference model $a_m = 6$ and $b_m = 9$ at adaptation gain 1, 2, 3, 4, and 5.

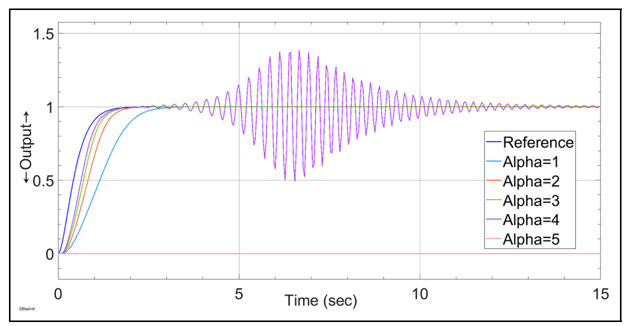

Figure 32. Simulation results of MRAC with Lyapunov rule for reference model $a_m = 8$ and $b_m = 16$ at adaptation gain 1, 2, 3, 4, and 5.

- From Figures 4 and 6, it is seen that system shows a good response till $\alpha = 100$, but from $\alpha = 100$ to 1000, some oscillations are introduced in the system for some time, but finally, it is settled with the reference model. After $\alpha = 1000$, oscillations start increasing and are not agreeing with the reference model.
- After this, the investigation of the performance concerning the change in the reference model is carried out. Optimal values of α have been selected for further analysis of change in reference model parameters, as shown in Figures 12–21 and Table 2. From these observations, it may be concluded that as the time constant of the reference model is increased, the system response is narrowed, and settling time is decreased with some overshoot values till $\alpha = 1000$ for the value of $a_m > a$, but for $a_m < a$, system's response is tracking to the reference model, but the settling time is increased with the increased value of α.
- Hence, it may be concluded that for the fist-order system, the adaptation gain should be in the range of 1–1000, and the reference model's time constant should be greater than the time constant of the first-order system.
- All the investigations are done for both MIT and Lyapunov rule separately, and comparative analysis

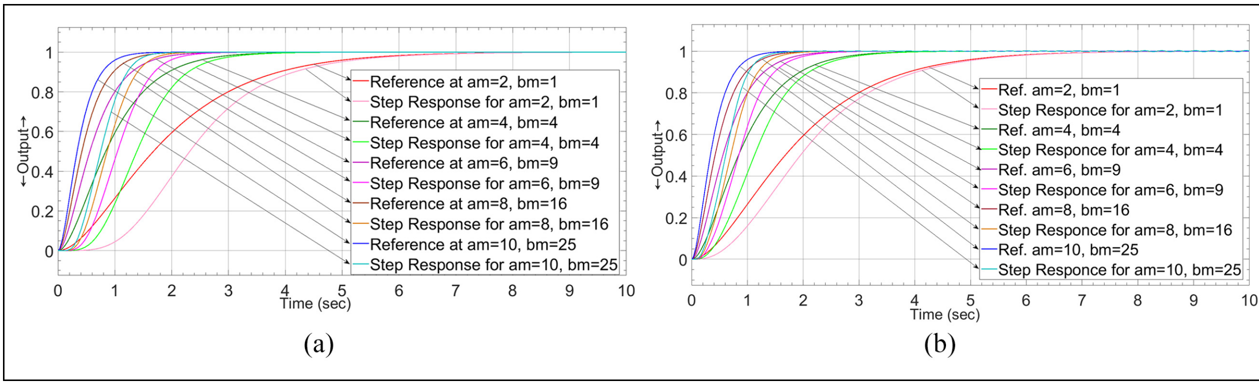

Figure 33. Step response of the second-order system for deferent references at α = 3 using (a) MIT rule and (b) Lyapunov rule.

Table 3. Effect of change of parameters of reference model on second-order system with MIT rule and Lyapunov rule.

Adaptation law	MIT rule					Lyapunov rule			
Reference model	Adaptation gain	Overshoot	Peak time	Settling time	Rise time	Overshoot	Peak time	Settling time	Rise time
$\frac{1}{s^2+2s+1}$	1	2.2204e−14	39.5939	6.6308	3.4372	0	25	6.2746	3.3573
	2	2.2204e−14	40.3386	6.1194	2.9700	0	25	6.0305	3.2267
	3	4.0424e−05	22.1758	6.0121	2.9044	0.0175	24.7899	5.9591	3.2372
	4	0	27.1262	27.1262	7.4607e−14	40.9613	12.1083	18.1199	3.2301
	5	0	14.0713	14.0713	3.9080e−14	52.2574	7.5366	12.6483	3.2413
$\frac{4}{s^2+4s+4}$	1	2.2204e−14	21.2969	4.1963	2.2972	2.2204e−14	20.3499	3.5964	2.0492
	2	3.4195e−12	16.9115	3.3111	1.7084	3.1308e−12	17.6792	3.1355	1.6721
	3	3.0037e−04	10.7617	3.1256	1.5411	0.0363	24.9530	3.1078	1.6264
	4	0	23.5345	23.5345	7.1054e−14	40.0135	9.2487	15.0174	1.6007
	5	0	12.6046	12.6046	3.5527e−14	57.9158	5.3611	10.9710	1.5890
$\frac{9}{s^2+6s+9}$	1	4.4409e−14	17.4461	3.4693	1.9394	2.2204e−14	15.0509	2.9018	1.6921
	2	1.0567e−10	10.7740	2.4793	1.3253	1.5219e−06	7.5762	2.2098	1.2161
	3	0.0013	13.7008	2.2083	1.1339	0.7768	19.8781	2.0266	1.0987
	4	0	19.2199	19.2199	5.3291e−14	39.7592	7.5318	13.7827	1.0707
	5	0	21.0657	4.5397	1.3253	52.8847	4.5322	9.5964	1.0592
$\frac{16}{s^2+8s+16}$	1	2.2204e−14	16.6686	3.1229	1.7682	2.2204e−14	14.3965	2.6221	1.5392
	2	2.1820e−05	5.1239	2.0944	1.1403	7.7983e−05	4.8487	1.7922	1.0127
	3	0.0086	11.8356	1.7853	0.9407	0.1771	14.7420	1.6471	0.8834
	4	0	16.9825	16.9825	4.6185e−14	37.6409	6.6590	13.0336	0.8447
	5	0	8.5337	8.5337	2.3093e−14	51.7270	3.6786	8.9262	0.8134

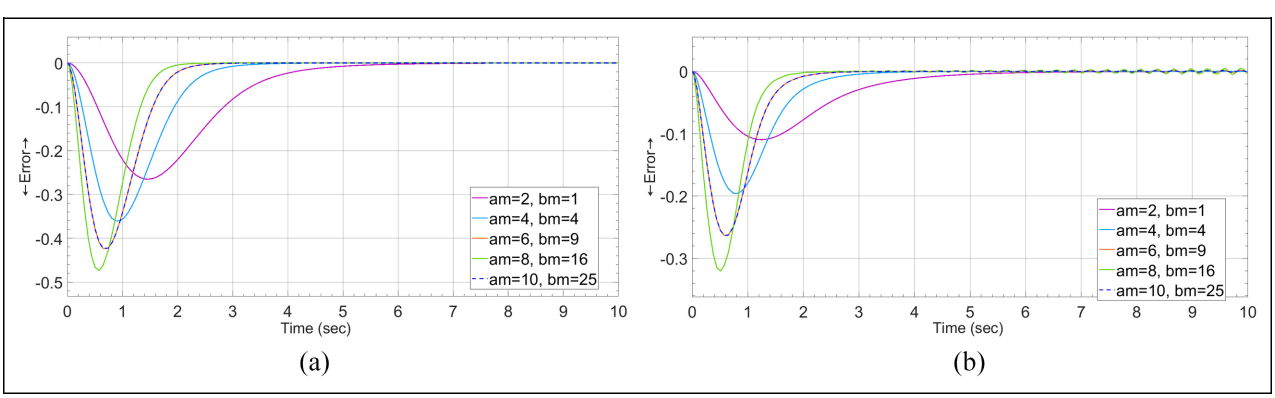

Figure 34. Output error of the second-order system for deferent references at α = 3 using (a) MIT rule and (b) Lyapunov rule.

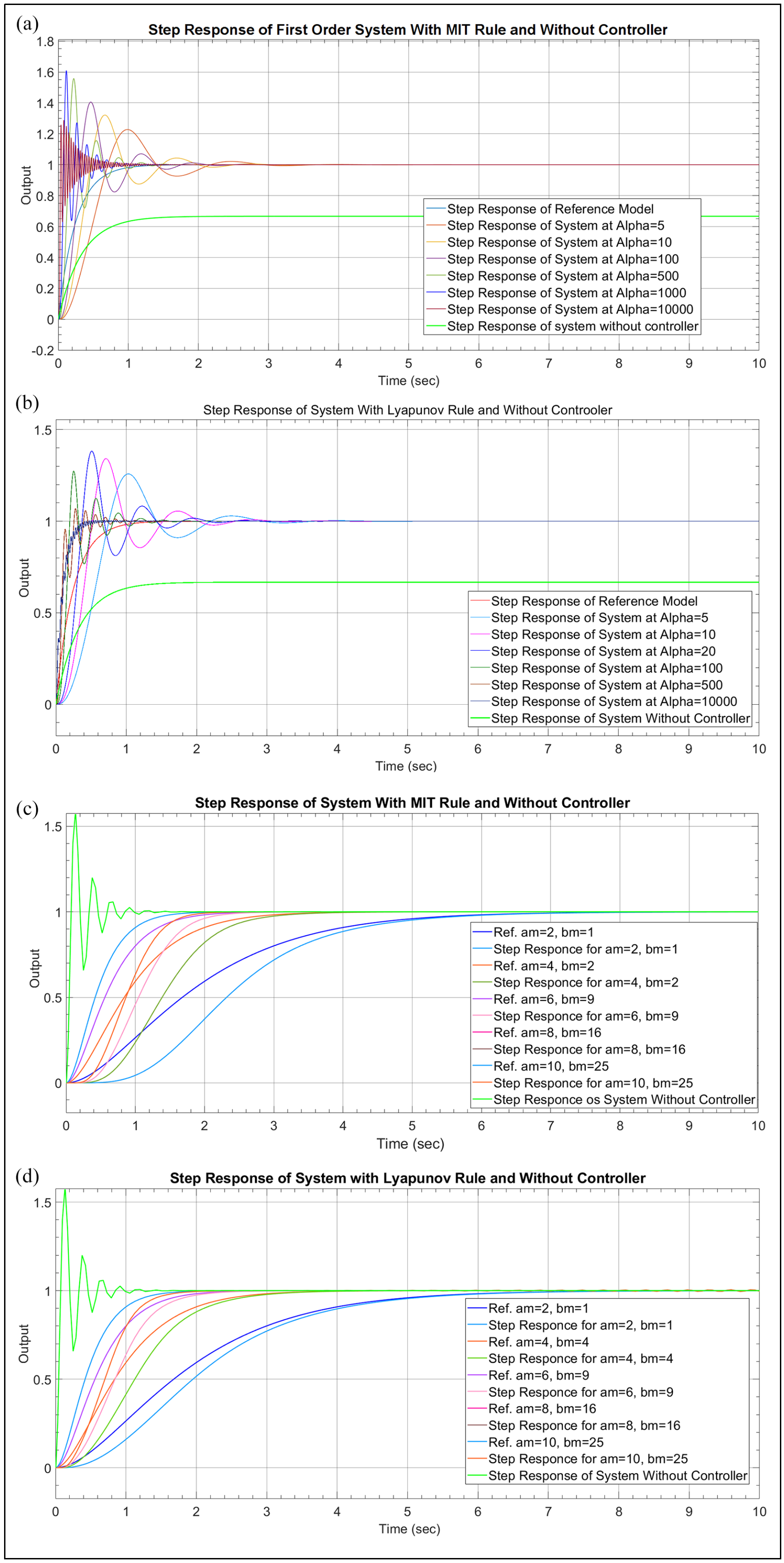

Figure 35 (a) Step response of the first-order system with and without controller (MIT rule-based). (b) Step response of the first-order system with and without controller (Lyapunov rule-based). (c) Step response of the second-order system with and without controller (MIT rule-based). (d) Step response of the second-order system with and without controller (Lyapunov rule-based).

suggests that design via the Lyapunov law performs superior to the MIT rule.

For second-order system

- For the second-order system, the investigations are done for adaptation gain and change in reference model parameters with the MIT and Lyapunov rule separately, shown in the results depicted in Figures 22–33 and Table 3.
- The settling time and overshoot of the system reduced with increasing natural frequency of the reference model for the range $\alpha = 1–3$ in both MIT and Lyapunov rules. However, the system became unstable for $\alpha = 4$ and 5 in the MIT rule and stable with minor oscillations in Lyapunov law for $\alpha = 4$ and 5.

Hence, it is concluded that α should be 1 to 3 for any second-order critically damped reference model.

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