भारतीय मानक

Indian Standard

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ISO 7870-2 : 2023

उत्पादन के दौरान सांख्यिकीय गुणता नियंत्रण की प्रणाली

भाग 10 शेव्हर्ट नियंत्रण चार्ट

Methods for Statistical Quality Control During Production

Part 10 Shewhart Control Charts

ICS 03.120.30

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NATIONAL FOREWORD

This Indian Standard (Part 10) which is identical to ISO 7870-2 : 2023 'Control charts — Part 2: Shewhart control charts' issued by the International Organization for Standardization (ISO) was adopted by the Bureau of Indian Standards on the recommendation of the Statistical Methods for Quality, Data Analytics and Reliability Sectional Committee and approval of the Management and Systems Division Council.

This Indian Standard is published in several parts. The other parts in this series are:

- Part 0 Guidelines for selection of control charts
- Part 1 Control charts for variables
- Part 2 Control charts for attributes
- Part 3 Specialized control charts
- Part 4 Cumulative sum chart
- Part 5 Acceptance control charts
- Part 6 EWMA control charts
- Part 8 Charting techniques for short runs and small mixed batches
- Part 9 Control charts for stationary processes

The text of the ISO standard has been approved as suitable for publication as an Indian Standard without deviations. Certain conventions are, however, not identical to those used in Indian Standards. Attention is particularly drawn to the following:

a) Wherever the words 'International Standard' appear referring to this standard, they should be read as 'Indian Standard'; and

b) Comma (,) has been used as a decimal marker while in Indian Standards, the current practice is to use a point (.) as the decimal marker.

In this adopted standard, reference appears to an International Standard for which Indian Standard also exists. The corresponding Indian Standard, which is to be substituted in its place, is listed below along with its degree of equivalence for the editions indicated:

International Standard	*Corresponding Indian Standard*	*Degree of Equivalence*
ISO 3534-2 Statistics — Vocabulary and symbols — Part 2: Applied Statistics	IS 7920 (Part 2) : 2012/ISO 3534-2 : 2006 Statistics — Vocabulary and symbols: Part 2 Applied statistics (*third revision*)	Identical

Annex A and Annex B are for information only.

For the purpose of deciding whether a particular requirement of this standard is complied with, the final value, observed or calculated, expressing the result of analysis shall be rounded off in accordance with IS 2 : 2022 'Rules for rounding off numerical values (*second revision*)'. The number of significant places retained in the rounded off value should be the same as that of the specified value in this standard.

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Introduction

A traditional approach to manufacturing has been to depend on production to make the product and on quality control to inspect the final product and screen out items not meeting specifications. This strategy of detection is often wasteful and uneconomical because it involves after-the-event inspection when the wasteful production has already occurred. Instead, it is much more effective to institute a strategy of prevention to avoid waste by not producing unusable output in the first place. This can be accomplished by gathering process information and analysing it so that timely action can be taken on the process itself.

Dr. Walter Shewhart in 1924 developed the control chart method for controlling the quality during production. Control chart theory recognizes two kinds of variability. The first kind is random variability (also known as natural/inherent/uncontrollable variation) arising due to causes known as chance/common/random causes. This is due to the wide variety of causes that are consistently present and not readily identifiable, each of which constitutes a very small component of the total variability but none of them contributes any significant amount. Nevertheless, the sum of the contributions of all of these unidentifiable random causes is measurable and is assumed to be inherent to the process. The elimination or correction of common causes may well require a decision to allocate resources to fundamentally change the process and system.

The second kind of variability represents a real change in the process. Such a change can be attributed to some identifiable causes that are not an inherent part of the process and which can, at least theoretically, be eliminated. These identifiable causes are referred to as "assignable causes" (also known as special/unnatural/systematic/controllable causes) of variation. They may be attributable to such matters as the lack of uniformity in material, a broken tool, workmanship or procedures, the irregular performance of equipment, or environmental changes.

A process is said to be in a state of statistical control, or simply "in control", if the process variability results only from random causes. Once this level of variation is determined, any deviation from this level is assumed to be the result of assignable causes that should be identified and eliminated.

The major statistical tool used to do this is the control chart, which is a method of presenting and comparing information based on a sequence of observations representing the current state of a process against limits established after consideration of inherent process variability. The control chart method helps first to evaluate whether a process has attained, or continues in, a state of statistical control. When the process is deemed to be stable and predictable, then further analysis regarding the ability of the process to satisfy the requirements of the customer may be conducted. The control chart also can be used to provide a continuous record of a quality characteristic of the process output while process activity is ongoing. Control charts aid in the detection of unnatural patterns of variation in data resulting from repetitive processes and provide criteria for detecting a lack of statistical control. The use of a control chart and its careful analysis leads to a better understanding of the process and will often result in the identification of ways to make valuable improvements.

Indian Standard

METHODS FOR STATISTICAL QUALITY CONTROL DURING PRODUCTION

PART 10 SHEWHART CONTROL CHARTS

1 Scope

This document establishes a guide to the use and understanding of Shewhart control chart approach to the methods for statistical control of a process.

This document is limited to the treatment of statistical process control methods using only Shewhart system of charts. Some supplementary material that is consistent with Shewhart approach, such as the use of warning limits, analysis of trend patterns and process capability is briefly introduced. However, there are several other types of control charts which can be used in different situations.

2 Normative references

The following documents are referred to in the text in such a way that some or all of their content constitutes requirements of this document. For dated references, only the edition cited applies. For undated references, the latest edition of the referenced document (including any amendments) applies.

ISO 3534-2, *Statistics — Vocabulary and symbols — Part 2: Applied statistics*

3 Terms and definitions

3.1 General presence

For the purposes of this document, the terms and definitions given in ISO 3534-2 apply.

ISO and IEC maintain terminology databases for use in standardization at the following addresses:

— ISO Online browsing platform: available at https://www.iso.org/obp

— IEC Electropedia: available at https://www.electropedia.org/

3.2 Symbols

NOTE The ISO/IEC Directives make it necessary to depart from common SPC usage in respect to the differentiation between abbreviated terms and symbols. In ISO standards an abbreviated term and its symbol can differ in appearance in two ways: by font and by layout. To distinguish between abbreviated terms and symbols, abbreviated terms are given in Cambria upright and symbols in Cambria or Greek italics, as applicable. Whereas abbreviated terms can contain multiple letters, symbols consist only of a single letter. For example, the conventional abbreviation of upper control limit, UCL, is valid but its symbol in equations becomes U_{CL}. The reason for this is to avoid misinterpretation of compound letters as an indication of multiplication.

3.2.1 For the purposes of this document, the following symbols apply

n Subgroup size; the number of sample observations per subgroup

k Number of subgroups

L Lower specification limit

L_{CL} Lower control limit

L_{CLi} Lower control limit for i^{th} subgroup

CL Centre line

U_{CL} Upper control limit

U_{CLi} Upper control limit for i^{th} subgroup

X Measured quality characteristic (individual values are expressed as (X_1, X_2, X_3,...). Sometimes the symbol Y is used instead of X

$\bar{X}$ (X bar) Subgroup average

$\bar{\bar{X}}$ (X double bar) Average of the subgroup averages

μ True process mean

μ_0 A given or prespecified value of μ

σ True process standard deviation

σ_0 A given or prespecified value of σ

$\tilde{X}$ Median of a subgroup

$\bar{\tilde{X}}$ Average of the subgroup medians

R Subgroup range

$\bar{R}$ Average of subgroup ranges

R_m Subgroup moving range

$\bar{R}_m$ Average moving range

s Subgroup sample standard deviation

$\bar{s}$ Average of subgroup sample standard deviations

p Proportion of nonconforming items in a subgroup

$\bar{p}$ Average proportion of nonconforming items for all subgroups

np Number of nonconforming items in a subgroup

p_0 A given value of p

np_0 A given value of np (for a given p_0)

c Number of nonconformities in a subgroup

c_0 A given value of c

$\bar{c}$ Average number of nonconformities for all subgroups

u Number of nonconformities per unit in a subgroup

$\bar{u}$ Average number of nonconformities per unit

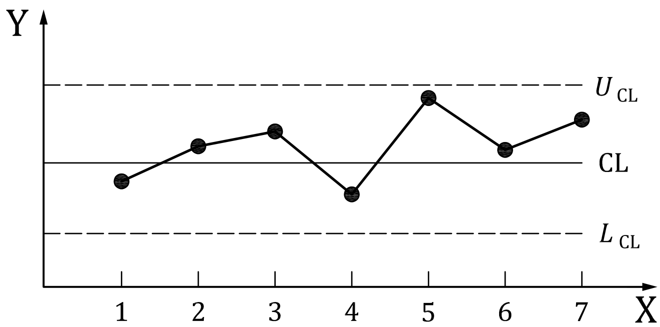
4 Concepts of Shewhart control charts

4.1 Shewhart control chart

A Shewhart control chart is a chart that is used to display a statistical measure (also called 'statistic') obtained from either variables or attribute data. The control chart requires data from rational subgroups (see 11.3) to be taken at approximately regular intervals from the process. The intervals may be defined in terms of time (for example hourly) or quantity (every lot). Usually, the data are obtained from the process in the form of samples or subgroups consisting of the same process characteristic, product or service with the same measurable units and the same subgroup size. From each subgroup, one or more statistical measures are calculated, such as average, $\bar{X}$, range, R, standard deviation, s, proportion of nonconforming items p, and number of nonconformities, c.

4.2 Control limits

Shewhart control chart is a chart on which some statistical measure of the values in each subgroup is plotted against subgroup number. It consists of centre line, CL, which is usually the average value of the statistical measure being considered or may be based on past experience, when the process is in state of statistical control. It may also be based on product or service target values. The control chart has two statistically determined limit lines, one on either side of the centre line, which are called the upper control limit, U_{CL}, and the lower control limit, L_{CL}, (see Figure 1).



Key

X subgroup number

Y statistic

CL centre line

L_{CL} lower control limit

U_{CL} upper control limit

Figure 1 — Outline of a control chart

4.3 Process in statistical control

4.3.1 The upper and lower control limits on the control chart, on each side of the centre line, are typically placed at a distance of three times the standard deviation of the statistic (3 σ) being plotted. If large number of observations from a process in statistical control are studied in form of frequency distribution, it often shows a bell shaped symmetrical pattern, which is well represented as normal distribution.

4.3.2 Placing the limits too close to the centre line will result in many searches for non-existing problems and yet placing the limits too far apart will increase the risk of not detecting process problems when they do exist. Under an assumption that the plotted statistic is approximately normally distributed 3 σ limits indicate that approximately 99,73 % of the values of the statistic will be included within the control limits, provided the process is in statistical control. Interpreted another way, there is a 0,27 % probability, or about three out of thousand plotted points will be out of the upper or lower control limit when the process is in control. The word "approximately" is used because deviations from underlying assumptions such as the distributional form of the data will affect the probability values. In fact, the choice of $k\ \sigma$ limits, instead of 3 σ limits, depends on costs of investigation and taking appropriate action vis-à-vis consequences of not taking action.

4.4 Action limits

The possibility that a violation of the limits is really a chance event rather than a real signal is considered so small that when a point appears outside of the limits, action should be taken. Since action is required at this point, the 3 σ control limits are sometimes called the "action limits".

4.5 Warning limits

Sometimes it is advantageous to mark 2 σ limits on the chart also. Then, any sample value falling beyond the 2 σ limits can serve as a warning of an impending out-of-control situation. As such, the 2 σ limits are sometimes called "warning limits". While no action is required as a result of such a warning on the control chart, some users may wish to immediately select another subgroup of the same size to determine if corrective action is needed.

4.6 Type 1 error

When assessing the status of a process using control charts, two types of errors are possible. The first occurs when the process is actually in a state of control but a plotted point falls outside the control limits due to chance (Type 1 error). As a result, the chart has given a false signal resulting in an incorrect conclusion that the process is out of control. A cost is then incurred in an attempt to find the cause of a non-existent problem.

If normality is assumed and 3 σ control limits are used, the probability of Type 1 error is 0,27 %. In other words, this error will happen only about 3 times in 1 000 samples when the process is in control.

4.7 Type 2 error

4.7.1 The second error occurs when the process involved is not in control but the plotted point falls within the control limits due to chance (Type 2 error). In this case, the chart provides no signal and it is incorrectly concluded that the process is in statistical control. There may also be a substantial cost associated with failing to detect that a change in the process location or variability has occurred, the result of which might be the production of nonconforming output. The risk of this type of error occurring is a function of three things: the width of the control limits, the sample size, and the degree to which the process is out of control. In general, because the magnitude of the change in the process cannot be known, little can be determined about the actual size of the risk of this error.

4.7.2 Because it is generally impractical to make a meaningful estimate of probability of Type 2 error in any given situation, Shewhart control chart system is designed to control the risk (or probability) of Type 1 error.

4.8 Process not in control

When a plotted value falls outside of either control limit, or a series of values display an unusual pattern such as discussed in Clause 8 and Annex B, the state of statistical control can no longer be accepted. When this occurs, an investigation is initiated to locate the assignable cause, and the process may be stopped or adjusted. Once the assignable cause is determined and eliminated, the process is ready to continue. As discussed in 4.3.2, on rare occasions when no assignable cause can be found and it must be concluded that the point outside the limits represents the occurrence of a rare event, a false signal, which has resulted in a value outside of the control limits even though the process is in control.

NOTE Point on the control line is considered as point in control.

4.9 Phase 1 of statistical process control

When a process is to be studied for the first time with the objective of bringing the process in a state of statistical control, it is often found necessary to use historical data that has previously been obtained from the process or to undertake to obtain new data from a series of samples before attempting to establish the control chart. This retrospective stage during which the control chart parameters are being established is often referred to as Phase 1. Sufficient data will need to be found in order to obtain reliable estimates of the centre line and control limits for the control charts. The control limits established in Phase 1 are trial control limits as they are based upon data collected when the process may not be in control. The identification of the precise causes for signals given by the control chart at this stage may prove to be difficult because of the lack of information about the historical operating characteristics of the process. However, when special causes of variation can be identified and corrective action taken, the retrospective data from the process when under the influence of the special cause should be removed from consideration and the control chart parameters re-determined. This iterative procedure is continued until the final trial control chart shows no signals and the control limits then correspond to the process in control. Because some data may have to be removed from consideration during Phase 1, some additional data may have to obtained from the process to maintain the reliability of the parameter estimates.

4.10 Phase 2 of control charts

Once statistical control has been established, the revise control limits in Phase 1 are taken as the control limits for the ongoing monitoring of the process. The objective now, in what is referred to as Phase 2, is the maintenance of the process in a state of control as well as the rapid identification of special causes that may affect the process from time to time. It should be recognized that moving from Phase 1 to Phase 2 may be time consuming and difficult. However, it is critical, because failure to remove special causes of variation will result in overestimation of the process variation. In this case the control chart will have control limits that are set too wide apart resulting in a control chart that is not sufficiently sensitive for detecting the presence of special causes.

5 Types of control charts

5.1 Types of Shewhart control charts

5.1.1 Shewhart control charts are of following two types:

a) variables control charts;

b) attribute control charts.

5.1.2 For each of these control charts, there are two distinct situations:

a) when no pre-specified process parameters values are given;

b) when pre-specified process parameters values are given.

5.2 Control charts where no pre-specified values of process parameters are given

The purpose is to identify whether the values of the statistics, which are being plotted on the control charts for different subgroups, differ from the centre line by an amount greater than that can be attributed to chance causes only. Control charts will be constructed using only the data collected from samples from the process. The control charts are used for detecting those variations caused other than by chance with the purpose being to bring the process in a state of statistical control.

5.3 Control charts with respect to given pre-specified values of process parameters

5.3.1 The purpose is to identify whether the observed values of $\overline{X}$, *s*, etc., for several subgroups of *n* observations each, differ from the respective given values of μ_0, σ_0, etc. by amounts greater than that expected to be due to chance causes only. The difference between charts with given parameter values and those where no pre-specified values are given, is the additional requirement concerning the determination of the location of the centre and variation of the process. The pre-specified values may be based on experience obtained by using control charts with no prior information or specified values. They may also be based on economic values established upon consideration of the need for service and cost of production or be nominal values designated by the product specifications.

5.3.2 Preferably, the specified values should be determined through an investigation of preliminary data that is supposed to be typical of all future data. The specified values should be compatible with the inherent process variability for effective functioning of the control charts. Control charts based on such pre-specified values are used particularly during process operation to control processes and to maintain product or service uniformity at the desired level.

5.4 Types of variables and attribute control charts

5.4.1 Variables control charts

The following control charts for variables are considered when measurements are on continuous scales:

a) average, $\overline{X}$ chart, and range, *R* chart, or standard deviation, *s* chart;

b) individuals, *X* chart and moving range, *R* chart;

c) median, $\tilde{X}$ chart and range, *R* chart.

5.4.2 Attribute control charts

The following attribute control charts are used when items are classified as conforming and nonconforming or number of nonconformities are counted on the items:

a) *p* chart for proportion of nonconforming items, when sample size is not constant;

b) *np* chart for number of nonconforming items when the sample size is constant.

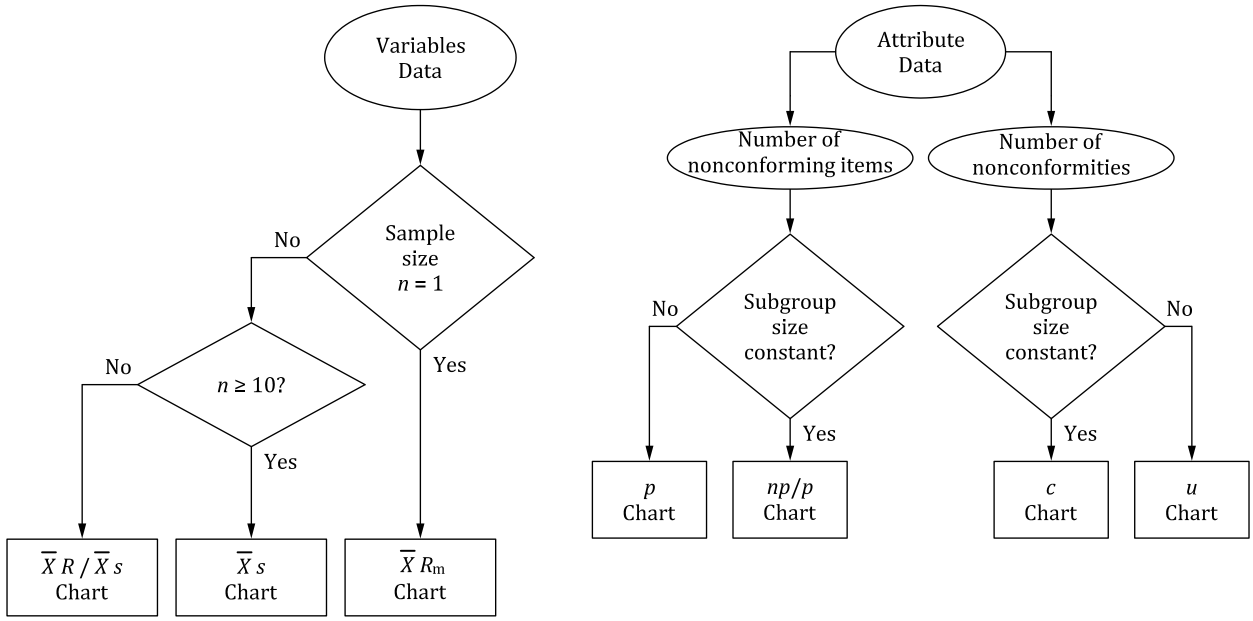
NOTE *p* chart can also be used in such a case. As it involves additional calculation to find *p* value for each subgroup for plotting them on *p* chart, and the result being the same as that of *np* chart; it is recommended to use *np* chart when sample size is constant.

c) *c* chart for number of nonconformities when the sample size is constant;

NOTE *u* chart can also be used in such a case. As it involves additional calculation to find *u* value for each subgroup for plotting them on *u* chart, and the result being the same as that of *c* chart; it is recommended to use *c* chart when sample size is constant.

d) *u* chart for the number of nonconformities per unit when the sample size is not constant.

Figure 2 shows a process of selecting an appropriate control chart for a given situation.



a) **Variables data** b) **Attribute data**

Figure 2 — Types of control charts

6 Variables control charts

6.1 Usefulness of variables control charts

Control charts for variables are particularly useful for several reasons including the following:

a) Most processes, and their output, have characteristics that are measurable, hence generate variables data, so the potential applicability is broad.

b) Variables charts are more informative than attribute charts since specific information about the process average and variation is obtained directly.

c) Although obtaining information on variables data for one item is more costly than that for attribute data, the subgroup sizes needed for variables data are smaller than those for attribute data, for an equivalent monitoring efficiency. This helps to reduce the total inspection cost and to shorten the time gap between the occurrence of a process problem and its corrective action.

d) Variables charts provide visual means to directly assess process performance regardless of the specifications.

6.2 Assumption of normality

For all variables control charts considered in this document, it is assumed that the distribution of the quality characteristic is normal. The factors used for computing control limits are derived using the assumption of normality. So, departure from this assumption will affect the performance of the charts. Since most control limits are used as empirical guides in making decisions, reasonably small departures from normality should not be of concern. In any case, because of the central limit theorem, averages tend to be normally distributed even when individual observations are not; this makes it reasonable for evaluating control to assume normality for $\overline{X}$ charts, even for sample sizes as small as 4 or 5. When dealing with individual observations for capability study purposes, the true form of the distribution is important. Periodic checks on the continuing validity of such assumptions are advisable, particularly for ensuring that only data from a single population are being used. It should be noted that the distributions of the ranges and standard deviations are not normal. Although normality is necessarily assumed in the determination of the constants for the calculation of control limits for the range or standard deviation chart, moderate deviations from normality of the process data should not be of major concern in the use of these charts as an empirical decision procedure.

6.3 Pair of control charts

6.3.1 As normality is assumed for variables type of data, and normal distribution has two parameters, namely, mean and standard deviation; a pair of control charts is prepared and analysed together, one for controlling variation of the process and the other for process mean. So, variables charts can describe process data in terms of both process variability (spread) and process average (location). Average, $\overline{X}$ chart is commonly used to control location and range, R chart to control inherent variability.

6.3.2 Each chart can be plotted using either estimated control limits, in which case limits are based on the information contained in the sample data plotted on the chart, or pre-specified control limits based on adopted specified values applicable to the statistical measures plotted on the chart.

6.3.3 The chart for spread is analysed first, since it provides the rationale and justification for the estimation of the process standard deviation. The resulting estimate of the process standard deviation is then be used in establishing control limits for the chart for location.

6.4 Average, $\overline{X}$ chart and range, R chart or average, $\overline{X}$ chart and standard deviation, s chart

$\overline{X}$ and R control charts can be used when subgroup sample size is small or moderately small, usually less than 10. $\overline{X}$ and s control charts are preferable in the case of large subgroup sample sizes ($n \geq 10$), since the range becomes increasingly less efficient in estimating the process standard deviation when the sample size gets larger. Where software is available to calculate process limits, standard deviation chart is preferable. Table 1 and Table 2 give the control limit formulae and the factors for each of these variables control charts.

Table 1 — Control limit formulae for average, range and standard deviation

Statistic	Estimated control limits		Pre-specified control limits	
	Centre line	U_{CL} and L_{CL}	Centre line	U_{CL} and L_{CL}
$\overline{X}$	$\overline{\overline{X}}$	$\overline{\overline{X}} \pm A_2\overline{R}$ and $\overline{\overline{X}} \pm A_3\overline{s}$	μ_0	$\mu_0 \pm A\sigma_0$
R	$\overline{R}$	$D_4\overline{R}, D_3\overline{R}$	$d_2\sigma_0$	$D_2\sigma_0, D_1\sigma_0$
s	$\overline{s}$	$B_4\overline{s}, B_3\overline{s}$	$c_4\sigma_0$	$B_6\sigma_0, B_5\sigma_0$
NOTE μ_0 and σ_0 are given values of parameters.				

Table 2 — Factors for computing control chart lines

subgroup size *n*	Factors for control limits											Factors for centre line	
	$\overline{X}$ chart			*s* chart				*R* chart[a]				Using *s*	Using *R*[a]
	A	A_2	A_3	B_3	B_4	B_5	B_6	D_1	D_2	D_3	D_4	c_4	d_2
2	2,121	1,880	2,659	0	3,267	0	2,606	0	3,686	0	3,267	0,798	1,128
3	1,732	1,023	1,954	0	2,568	0	2,276	0	4,358	0	2,575	0,886	1,693
4	1,500	0,729	1,628	0	2,266	0	2,088	0	4,698	0	2,282	0,921	2,059
5	1,342	0,577	1,427	0	2,089	0	1,964	0	4,918	0	2,114	0,940	2,326
6	1,225	0,483	1,287	0,030	1,970	0,029	1,874	0	5,079	0	2,004	0,952	2,534
7	1,134	0,419	1,182	0,118	1,882	0,113	1,806	0,205	5,204	0,076	1,924	0,959	2,704
8	1,061	0,373	1,099	0,185	1,815	0,179	1,751	0,388	5,307	0,136	1,864	0,965	2,847
9	1,000	0,337	1,032	0,239	1,761	0,232	1,707	0,547	5,394	0,184	1,816	0,969	2,970
10	0,949	0,308	0,975	0,284	1,716	0,276	1,669	0,686	5,469	0,223	1,777	0,973	3,078
11	0,905	0,285	0,927	0,321	1,679	0,313	1,637	0,811	5,535	0,256	1,744	0,975	3,173
12	0,866	0,266	0,886	0,354	1,646	0,346	1,610	0,923	5,594	0,283	1,717	0,978	3,258
13	0,832	0,249	0,850	0,382	1,618	0,374	1,585	1,025	5,647	0,307	1,693	0,979	3,336
14	0,802	0,235	0,817	0,406	1,594	0,399	1,563	1,118	5,696	0,328	1,672	0,981	3,407
15	0,775	0,223	0,789	0,428	1,572	0,421	1,544	1,203	5,740	0,347	1,653	0,982	3,472
16	0,750	0,212	0,763	0,448	1,552	0,440	1,526	1,282	5,782	0,363	1,637	0,984	3,532
17	0,728	0,203	0,739	0,466	1,534	0,458	1,511	1,356	5,820	0,378	1,622	0,985	3,588
18	0,707	0,194	0,718	0,482	1,518	0,475	1,496	1,424	5,856	0,391	1,609	0,985	3,640
19	0,688	0,187	0,698	0,497	1,503	0,490	1,483	1,489	5,889	0,404	1,596	0,986	3,689
20	0,671	0,180	0,680	0,510	1,490	0,504	1,470	1,549	5,921	0,415	1,585	0,987	3,735
21	0,655	0,173	0,663	0,523	1,477	0,516	1,459	1,606	5,951	0,425	1,575	0,988	3,778
22	0,640	0,167	0,647	0,534	1,466	0,528	1,448	1,660	5,979	0,435	1,565	0,988	3,819
23	0,626	0,162	0,633	0,545	1,455	0,539	1,438	1,711	6,006	0,443	1,557	0,989	3,858
24	0,612	0,157	0,619	0,555	1,445	0,549	1,429	1,759	6,032	0,452	1,548	0,989	3,895
25	0,600	0,153	0,606	0,565	1,435	0,559	1,420	1,805	6,056	0,459	1,541	0,990	3,931

[a] Not recommended for sample size $n \geq 10$.

6.5 Control chart for individuals, *X*, and moving ranges, R_m

6.5.1 In these charts, only one sample from each subgroup is drawn. This is applicable in situations, where it is either impossible or impractical to draw more samples, like, the characteristic to be controlled is destructive and the item being very costly. In some situations, where the material is homogeneous (liquid or powder form), it does not make sense to have rational subgroups of size more than one. In such situations, only one sample from each subgroup will be sufficient. It is then necessary to assess process control based on individual readings using *X* and R_m charts.

6.5.2 In the case of control charts for individuals, since each subgroup is of size one, it will not be able to provide an estimate of variability within each subgroup. Hence, a measure of variation is obtained from moving ranges of two consecutive observations. A moving range is the absolute value of the difference between two successive measurements; i.e. the absolute value of the difference between the first and second measurements, then between the second and third, and so on. From the moving ranges, the average moving range is calculated and used for the construction of control charts. Also, from the entire collection of data, the overall average $\overline{X}$ is calculated. Table 3 gives the control limit formulae for control charts for individuals and moving ranges.

6.5.3 Some caution should be exercised with respect to control charts for individuals:

a) The charts for individuals are not as sensitive to process changes as charts based on subgroups with sample size more than one.

b) Care shall be taken in the interpretation of charts for individuals if the process distribution is not normal.

c) Charts for individuals isolate process variability from an average of consecutive differences between observations. Thus, it is implied that the data are time-ordered, and that no significant changes have occurred in the process in between the collection of any two consecutive individuals. It would be ill advised, for example, to gather data from two discontinuous campaigns of production of a batch chemical product and to calculate a moving range between the last batch of the first campaign and the first batch of the next campaign, if the production line has been stopped in between.

Table 3 — Control limit formulae for control charts for individuals

Statistic	No standard value given		Standard value given	
	Centre line	U_{CL} **and** L_{CL}	**Centre line**	U_{CL} **and** L_{CL}
Individual, X	$\overline{X}$	$\overline{X} \pm 2{,}660\overline{R}_m$	μ_0	$\mu_0 \pm 3\sigma_0$
Moving Range, R_m	$\overline{R}_m$	$3{,}267\overline{R}_m$, 0	$1{,}128\sigma_0$	$3{,}686\sigma_0$, 0
NOTE 1 μ_0 and σ_0 are pre-specified values.				
NOTE 2 $\overline{R}_m$ denotes the average of moving ranges of 2 consecutive observations.				

6.6 Control charts for medians, $\widetilde{X}$

6.6.1 Median charts are alternatives to $\overline{X}$ charts for the control of a process location when it is desired to reduce the influence of the extreme values in a subgroup. This might be the case for subgroups made of many automated measurements such as when measuring tensile strength. Median charts are easy to use and do not require as many calculations, particularly for subgroups of small size containing an odd number of observations. This can increase shop floor acceptance of the control chart approach. The chart then also shows the spread of process output. It should be noted that the median chart gives a marginally slower response to out-of-control conditions than the $\overline{X}$ chart.

6.6.2 Control limits for median charts are calculated in two ways: by using the median of the subgroup medians and the median of the ranges; or by using the average of the subgroup medians and the average of the ranges. Only the latter approach, which is easier and more convenient, is considered in this document.

6.6.3 The control limits for median chart are calculated as follows.

Median chart

$$\mathrm{CL} = \overline{\widetilde{X}} = \text{average of the subgroup medians}$$

$$U_{\mathrm{CL}\,\overline{\widetilde{X}}} = \overline{\widetilde{X}} + A_4\,\overline{R}$$

$$L_{\text{CL}\bar{\bar{X}}} = \bar{\bar{X}} - A_4 \bar{R}$$

The values of the constant A_4 are given in Table 4.

Table 4 — Values of A_4

n	2	3	4	5	6	7	8	9	10
A_4	1,880	1,187	0,796	0,691	0,548	0,508	0,433	0,412	0,362

6.6.4 Control limits for range chart

The range chart is constructed in the same way as for the case of the $\bar{X}$ and R chart in 6.4.

7 Control procedure and interpretation for variables control charts

7.1 Underlying principle

Shewhart system of charts stipulates that if the process location and the process variability are to remain constant at their present levels, the individual plotted statistics (e.g. $\bar{X}$, R, s) would vary by chance causes alone and they would seldom fall outside the control limits. Likewise, there would be no obvious trends or patterns in the data, beyond what would occur due to chance causes. The charts for location show changes in subgroup averages and indicate whether or not the process location is stable with respect to the average. The $\bar{X}$ chart, for example, reveals between-subgroups variations over time and is designed for detecting shifts in averages between the subgroups. The s or R chart reveals within-subgroup variation at a given time and is designed for detecting changes in process inherent variation. The s or R chart shall be in control before a location chart is interpreted. The following control procedure applies to the $\bar{X}$ and s (or R) charts. A similar procedure can be used for other control charts including the individual, X, chart.

7.2 Collect preliminary data

Gather preliminary rational subgroups (see 11.3) from a process under standard operating conditions. Compute the s (or R) of each subgroup. Compute the average ($\bar{s}$ or $\bar{R}$) of each subgroup. Typically, a minimum of 25 preliminary subgroups are taken to ensure reliable estimates ($\bar{s}$ or $\bar{R}$) of the process variability and consequently the control limits.

7.3 Examine s (or R) chart

Compute and plot the trial centre line and control limits of the s (or R) chart. Examine if any of the s (or R) value falls outside the upper control limit or for unusual patterns or trends. For each such point, conduct an analysis to identify assignable cause(s) and exclude all such subgroups for which assignable causes are identified.

NOTE 1 The sampling distributions of s and R are both asymmetric about their average values. A lower control limit of 0 is used when the calculated lower limit is a negative value.

NOTE 2 If one fails to identify an assignable cause for a point that is out-of-control, one should retain the point in the calculation of control limits.

7.4 Homogenization for s (or R) chart

Exclude all subgroups affected by the identified assignable causes; then recalculate the revised centre line and control limits. Examine the chart to determine if all remaining data points show statistical control when compared to the revised limits; repeat the identification and recalculation sequence if necessary. Once the standard deviations (or ranges) for all subgroups are in statistical control, the process variability (the within-subgroup variation) is considered to be stable. It is fixed now and not to be changed.

NOTE Ensure that at least 80 % of subgroups remain. Collect additional subgroups if necessary.

7.5 Homogenization for $\bar{X}$ chart

7.5.1 For computing control limits for $\bar{X}$ chart, the subgroups which are excluded for the construction of s (or R) chart shall also be excluded. For the remaining subgroups, compute the centre line and control limits of the $\bar{X}$ chart. Examine whether average for each subgroup falls outside the upper or lower control limits or shows any unusual patterns or trends. For all such out-of-control subgroups, identify assignable causes and exclude all such subgroups for which assignable causes are identified. Recalculate control limits for remaining subgroups. Check whether all data points are now within control limits when compared with the revised limits. This process of exclusion of subgroups, for which the data point falls outside control limits, is continued until all subgroups fall within control limits. When all points fall within control limits, then the process (without excluded subgroups) is considered to be in statistical control, namely, variation in the process is due to the chance causes only.

7.5.2 Out of control situations eliminated to determine control limits must not be excluded on the plotted chart to provide vital clues to know the process behaviour and aid investigations.

7.5.3 The control limits for variation (s or R) once finalized during homogenization process for variation are then fixed, and are not altered further due to exclusion of some subgroups for homogenization of averages.

7.5.4 If during homogenization process for average and range, more than 20 % of the subgroups are excluded, then the data itself is not considered as appropriate for computing control limits. Investigate, identify assignable cause(s) for all out-of-control points and remove them. Thereafter again collect fresh data for minimum 25 subgroups.

7.6 Ongoing monitoring of process

7.6.1 When statistical control has been established so that there are no out of control points on both average and s (or R) charts, these control limits shall then be adopted for future ongoing monitoring of the process. Because the process has been demonstrated to be in a state of statistical control, there is no need to alter the control limits as and when additional subgroups are obtained in this monitoring phase. However, one may wish to update the control limits from time to time or whenever there is any change in the process.

7.6.2 In the event of an out-of-control signal being given on the chart and assignable cause(s) identified, the elimination of which required substantial changes be made to the process, it is possible or likely that the procedure of recalculation outlined in 7.2 to 7.5 may be required to re-establish control limits of the process.

8 Unnatural pattern and tests for assignable causes of variation

8.1 Natural pattern

Since the basic aim of the control chart is to detect the presence of any unnatural pattern in the process, which will have to be removed, it is useful to have an idea about the following characteristics of a natural pattern of a process:

a) majority of the points are near the centre line;

b) a few of the points are spread out and approach the control limits;

c) none of the points (or at least only a rare and occasional point) exceeds the control limit.

8.2 Unnatural patterns

8.2.1 General

There are different types of unnatural patterns which may be noticed in the control chart. The ease and the frequency with which an operator will be able to spot any unnatural pattern will depend on his experience in running the control chart and his knowledge of the process. However, the following are some of the unnatural patterns, which occur more often:

a) Instability: the presence of points outside the control limits.

b) Stratification: up and down variations are very small in comparison with the width of the control limits and absence of points near the control limits.

c) Mixture: tendency to avoid the centre line with too many points near the control limits.

d) Cyclic pattern: a long series of points which are high, low, high, low without any interruption in this regular sequence.

e) Trend: a series of consecutive points (minimum seven) without a change in direction.

The above unnatural pattern may be observed if there is lack of control in:

a) average (or median) chart only,

b) range (or standard deviation) chart only, and

c) both the average and range charts.

8.2.2 Lack of control in the average chart only

This is the common type of lack of control observed in manufacturing wherein a shift in the process average occurs with little or no changes in the process dispersion. In such cases the control chart is often of great value to the machine setter to help him to centre the machine setting in order to produce a desired process average. This type of lack of control is shown on the average chart. Unless the changes in the process average take place within a subgroup, the range chart will show control. Since the control limits are set far enough from the centre line on the chart with the possibility of very few points outside the control limits without a real change in the process, small shifts in the process average will not cause many points to fall out of control. Sufficient grounds exist for suspicion that the process average has shifted when:

a) 9 consecutive points on the control chart are on the same side of the centre line,

b) 10 out of 11 consecutive points are on the same side of the centre line,

c) 12 out of 14 consecutive points are on the same side of the centre line,

d) 14 out of 17 consecutive points are on the same side of the centre line, and

e) 16 out of 20 consecutive points are on the same side of the centre line.

8.2.3 Lack of control in the variation chart only

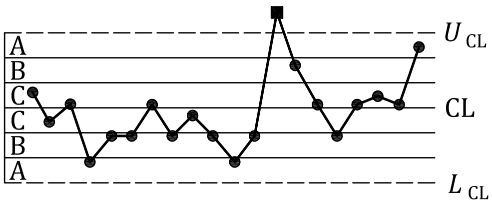
The inherent variability of a process may change from time-to-time even though there is no change in the process average. For any process where the skill and care of the operator is an important factor, the common cause of increase in variability is a change from one operator to another who is less skilful or less careful. In fact, an operator’s skill and care may also vary from day-to-day or from hour-to-hour. Extreme runs above the centre line on the range chart also give strong evidence of lack of control in the process variability. Generally speaking, variability of a process variation is particularly likely to be found in those processes where the skill of the operator is important. Hence the first step in improving such processes should be an attempt to bring the process dispersion in statistical control. The value of the average range, $\overline{R}$, used for the calculation of the control limits should be reviewed from time-to-time since it has a direct bearing on the calculation of the control limits for both the central tendency and dispersion.

8.2.4 Lack of control in both average and variation charts

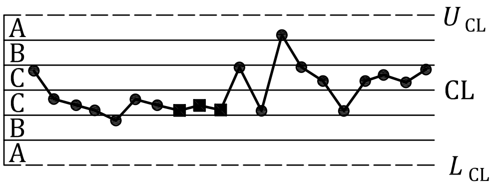
This lack of control in both average and variation charts is generally found in the initial stages of setting up of control charts. Where several assignable causes of variation exist, the elimination of some of the causes will decrease the number of out-of-control points but will not eliminate all of them. In such circumstances, one should not be discouraged by the inclusion of some points out of control. On the other hand, the chart should be viewed as an indication that further improvement is possible and as an incentive to keep hunting for more sources of trouble. It is also worthwhile examining whether an error in measurement may be an assignable cause of variation in the values resulting from the measurements, because an error in setting a measuring device may make apparent sudden shifts in process average. Frequent errors in setting(s) may also make irregular shifts in the average. Some type of wear of measuring device may cause increase in the process dispersion. Yet other types of wear may give rise to trends in averages.

8.2.5 Depiction of unnatural patterns

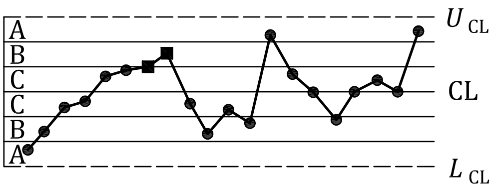
These unnatural patterns are depicted in Figure 3. However, the unnatural patterns should be regarded as guidelines, and not rules. There may be some processes, which as a natural pattern, exhibit any of the above unnatural patterns. For example, in oxidation process in semiconductor devices under influence of atmospheric pressure, runs are likely to appear in control charts. But such a state is not considered as unusual.



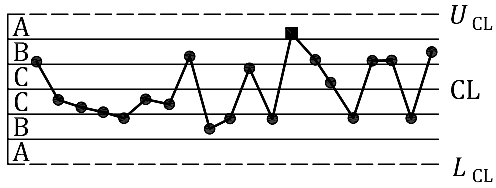
a) Example 1: one or more points are beyond zone A (outside the control limits)



b) Example 2: run – seven or more consecutive points on one side of centre line



c) Example 3: trend – seven consecutive points entirely increasing or decreasing



d) Example 4: non-random systematic pattern

Key

CL centre line

L_{CL} lower control limit

U_{CL} upper control limit

Figure 3 — Examples of pattern tests for assignable causes

8.2.6 For the purpose of applying these tests, the control chart is equally divided into three zones A, B, and C on each side of the centre line, each zone being one σ wide. This partitioning makes it easy for an investigator to detect a pattern that deviates away from a stable process. For example, the "non-random patterns" of Example 4 can be more easily detected when such partitions are applied. It is expected that about 2/3 of the plotted points to lie in zone C in a stable process. If substantially fewer than 2/3 of the plotted points lie in zone C, as shown in the Example 4 of Figure 3, one should be concerned about such a non-random pattern in the plot. Such a pattern calls for further investigation of their process for potential assignable causes. Following are the typical signals provided by the four examples in Figure 3:

a) Example 1 signals the presence of an out-of-control condition.

b) Example 2 signals the process average has shifted from the centre line.

c) Example 3 signals a systematic linear trend in the process.

d) Example 4 signals a non-random pattern in the process.

8.2.7 A process with a sequence of points on the chart that violates one or more of the rules is said to be out-of-control and its assignable causes of variation should be diagnosed and corrected. These supplementary rules do improve the ability of the control chart to detect smaller shifts in process average, but at the expense of higher false alarm rate. In Phase 2, increasing the false alarm rate should be avoided.

For a more complete discussion of these tests, see Annex B.

9 Process control, process capability, and process improvement

9.1 Process control

The function of a process control is to provide statistical signals separating non-assignable from assignable causes of variation, which leaves only non-assignable variations present. The systematic elimination of assignable causes of excessive variation through continuous determined efforts of eliminating causes brings the process into a state of statistical control. Once the process is operating in statistical control, its performance is predictable and its capability to meet the specifications can be assessed. Since prediction is the essence of management, this ability to know what to expect is invaluable in terms of the process operating more consistently, more predictably and more reliably.

9.2 Process capability and improvement

9.2.1 Process capability represents the performance of the process itself, as demonstrated when the process is being operated in a state of statistical control, see ISO 22514 (all parts). As such, the process shall first be brought into statistical control before its capability can be assessed. Thus, the assessment of process capability begins after control issues in both the $\overline{X}$ and R charts have been resolved; that is, special causes have been identified, analysed, corrected and prevented from occurring or recurring and the ongoing control charts reflect a process that has remained in statistical control, preferably for at least the past 25 subgroups. In general, the distribution of the process output is compared with the engineering specifications to see whether these specifications can consistently be met.

9.2.2 Process capability is generally measured in terms of a process capability index C_{p} and C_{pk}, see ISO 22514 (all parts). A C_{p} value of less than 1 indicates that the process is not capable, while a $C_{\text{p}} = 1$ implies that the process is only just capable. For C_{p} value lying in between 1 and 1,33, the process is capable. For C_{p} value beyond 1,33, the process is more than capable In practice, a C_{p} value of 1,33 is generally taken as the minimum acceptable value because there is always some sampling variation and few processes may not be in statistical control consistently.

9.2.3 However, it must be noted that the C_{p} measures only the relationship of the limits to the process spread; the location or the centring of the process is not considered. In that case, it is possible to have high percentage of values outside the specification limits even with a high C_{p} value. For this reason, it is important to consider the scaled distance between the process average and the closer specification limit. This property is measured or characterized by C_{pk}.

9.2.4 A procedure, as schematically presented in Figure 4, may be used as a guide to illustrate key steps leading towards process control, process capability and improvement.

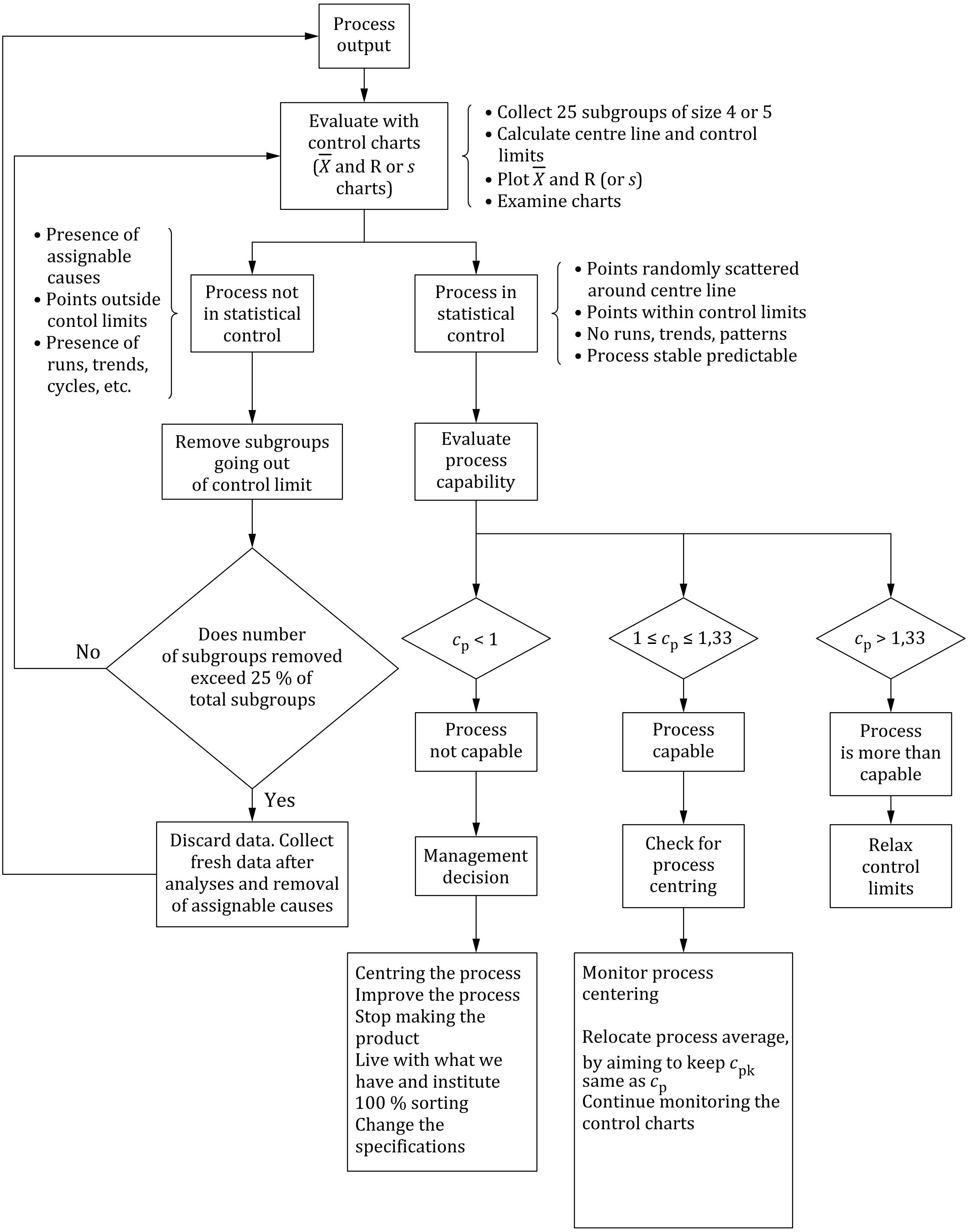


Figure 4 — Strategy for process improvement

10 Attribute control charts

10.1 Attribute data

10.1.1 Attribute data represent observations obtained by noting the presence or absence of some characteristic (or attribute) in each of the items in the subgroup under consideration, then counting how many items do or do not possess the attribute, or how many such events occur in the item, group or area. Attribute data are generally rapid and inexpensive to obtain and often do not require specialized collection skills. Table 5 gives control limit formulae for attribute control charts.

10.1.2 There is much attention focused on the use of variables data for process improvement, but feedback data from major industries indicate that over 80 % of quality problems are attribute in nature. More emphasis, therefore, is needed on the improvement of attribute characteristics using control charts.

10.2 Distributions

In the case of control charts for variables, it is common practice to maintain a pair of control charts – one for the control of the average and the other for the control of the dispersion. This is necessary because the underlying distribution in the control charts for variables is the normal distribution, which depends on the two parameters, namely mean and standard deviation. However, in the case of control charts for attribute, a single chart will suffice since the assumed distribution has only one independent parameter, the average level. The p and np charts are based on the binomial distribution, while the c and u charts are based on the Poisson distribution.

10.3 Subgroup size

10.3.1 Computations for these charts are similar except in cases where the variability in subgroup size affects the situation. When the subgroup size is constant, the same set of control limits can be used for each subgroup. However, if the number of items inspected in each subgroup varies, separate control limits have to be computed for each subgroup. np and c charts may thus be used with a constant sample size, whereas p and u charts may be used in either situation.

10.3.2 Where the sample size varies from subgroup to subgroup, separate control limits are calculated for each subgroup. The smaller the subgroup size, the wider the control bands, and vice versa. If the subgroup size does not vary appreciably, then a single set of control limits based on the average subgroup size may be used. For practical purposes, this holds well for situations in which the subgroup size is within ±25 % of the average subgroup size.

NOTE Alternatively, control limits for the two subgroups corresponding to the smallest and largest sample sizes may be calculated and plotted. The control limits for the largest sample size will be nearest to the centre line; and for the smallest sample size, farthest from the centre line. For other sample sizes, their control limits will lie in between these limits. Hence, for any subgroup, if its point lies within the control limits for the largest sample size, it is in control; and if the point falls outside the control limit for smallest sample size, the subgroup is out of the control limits. Only for subgroup points falling in between these two control limits, their control limits are required to be calculated.

Table 5 — Control limit formulae for attribute control charts

Statistic	No standard values given		Standard values given	
	Centre line	control limits	Centre line	control limits
p	$\bar{p}$	$\bar{p} \pm 3\sqrt{\bar{p}(1-\bar{p})/n}$	p_0	$p_0 \pm 3\sqrt{p_0(1-p_0)/n}$
np	$n\bar{p}$	$n\bar{p} \pm 3\sqrt{n\bar{p}(1-\bar{p})}$	np_0	$np_0 \pm 3\sqrt{np_0(1-p_0)}$
c	$\bar{c}$	$\bar{c} \pm 3\sqrt{\bar{c}}$	c_0	$c_0 \pm 3\sqrt{c_0}$
u	$\bar{u}$	$\bar{u} \pm 3\sqrt{\bar{u}/n}$	u_0	$u_0 \pm 3\sqrt{u_0/n}$
NOTE 1 p_0, np_0, c_0 and u_0 are given standard values.				
NOTE 2 A lower control limit of 0 is used when the calculated lower limit is a negative value.				

10.4 Control chart for fraction nonconforming (p chart)

10.4.1 The p chart is used to check whether fraction of nonconforming items of various subgroups is in control and then to determine the average fraction of nonconforming items submitted over a period of time. This information may be used by process personnel or the management to bring about any changes in the system. The process is judged to be in statistical control in the same way as is done for the $\bar{X}$ and R control chart. If all the sample points fall within the trial control limits without exhibiting any indication of an assignable cause, the process is said to be in control. In such a case, the average fraction nonconforming, $\bar{p}$, is taken as the standard value for the fraction nonconforming, p_0.

10.4.2 Low results on the control charts (points below the lower control limits) should be treated differently to points above upper control limit. It is a welcome step and are indicative of improvement in the process. When a significant break through the L_{CL} occurs, it is important to understand the causes and to institutionalize the changes in the work standards. But, it shall be ensured that low values of average fraction nonconforming have actually taken place, and there is no inadvertent error or due to lower inspection standards.

11 Preliminary considerations before starting a control chart

11.1 Choice of critical to quality (CTQ) characteristics describing the process to control

The characteristics that critically affect the performance of the product, process, or service, and which add value to the customer should be identified at the quality planning stage. These characteristics, where variation is the significant factor of the process should be selected to have a decisive effect on product or service quality and to ensure the stability and predictability of the processes. These may be aspects directly related to evaluation of the performance of the process – for example, related to the environment, health, customer satisfaction – or a process parameter whose performance is vital in achieving the design intent. Control charts should be introduced during the early stage of process development to collect data and information about a new product and process feasibility to achieve process capability prior to production.

11.2 Analysis of the process

11.2.1 If possible, a detailed analysis of the process should be made to determine:

a) the type and location of causes that may give rise to irregularities;

b) the effect of the imposition of specifications limits;

c) the method and location of inspection;

d) all other pertinent factors that may affect the production process.

11.2.2 Analysis should also be performed to determine the stability of processes, the accuracy of testing equipment, the quality of the outputs of the processes, and the patterns of correlation between the types and causes of nonconformities. The conditions of operations are required to make arrangements to adjust the production process and equipment, if needed, as well as to devise plans for the statistical control of processes. This will help pinpoint the most optimal place to establish controls and identify quickly any irregularities in the performance of the process to allow for prompt corrective action.

11.3 Choice of rational subgroup

11.3.1 Since the central idea of Shewhart's control charts is to separate the variation due to assignable and non-assignable causes, it is evident that each sample should be representative of a homogeneous segment of the production flow. So, in the ideal condition the variation found in items within a subgroup should be due to chance causes whereas the variation found between subgroups should be ascribable to some assignable causes. The division of the production flow in such a manner that each portion yields a sample having this property is known as 'rational subgrouping'. A rational subgroup for example may be the output of a short time period since the variation in items manufactured close to each other in the time sequence are much more likely to represent chance fluctuations.

11.3.2 The formation of rational subgroups depends on some technical knowledge and familiarity with the process conditions and the conditions under which the data are taken. By identifying each subgroup with a time or a source, specific causes of trouble may be more readily traced and corrected, if advantageous. Inspection and test records given in the order in which the observations are taken provide a basis for subgrouping with respect to time. This is commonly useful in manufacturing where it is important to maintain the production cause system constant with time.

11.3.3 Whereas it is not possible to give exact instructions for the formation of rational subgroups that will cover all cases, a few illustrations may be helpful in this direction. Thus, if different machine settings influence the quality characteristic that is being studied, all the items in a single subgroup should come from the same setting. Again, if different batches of material have an effect, then all items in one subgroup should be from the same batch. Extending this, it may generally be advisable not to form subgroups such that a single subgroup will consist of items manufactured in different shifts, from components obtained from different sources, from different production lines, from different machines, moulds, operators, etc. In many situations a small sample taken in the order of production meets the principle of rational subgroups, since it is likely to represent the immediate state of the process at the time a sample is selected. However, it should be noted that this is not a universal recommendation. If one is taking a sample from a machine with multiple spindles or multiple positions or heads, then a series of consecutive items from the machine will not form a rational subgroup because of the variation between the different heads. For example, if a filling machine has six heads which simultaneously fill six consecutive containers in the production line, then every sixth item taken (and not the consecutive six items) from the process will form a rational subgroup, because the variations within such subgroups would be the inherent variation due to the heads and the variation between the subgroups would be the variation obtaining from the different heads of the machine. In such situations, precise setting of the six heads becomes crucial.

11.3.4 In collecting data it should always be remembered that analysis will be greatly facilitated if care is taken to select the samples that can be properly treated as separate rational subgroups. If possible, the subgroup size should be kept constant to facilitate calculations and interpretation. However, it should be noted that the principles of Shewhart control charts can equally be applied to situations where subgroup size varies.

11.4 Frequency and size of subgroups

11.4.1 No general rules may be laid down for the frequency of subgroups or the subgroup size. The frequency and size of subgroup may depend upon the cost of taking and analysing samples and allied practical considerations. For instance, large subgroups taken at less frequent intervals may detect a small shift in the process average more accurately, but small subgroups taken at more frequent intervals will detect a large shift more quickly. Often, the subgroup size is taken to be 4 or 5, while the sampling frequency is generally high in the beginning and low once a state of statistical control is reached. Normally, 25 subgroups of size 4 or 5 are considered adequate for providing preliminary estimates.

11.4.2 It is worth noting that sampling frequency, statistical control and process capability need to be considered together. The reasoning is as follows. The value of the average range, $\bar{R}$, is often used to estimate σ. The number of sources of variation increases as the time interval between samples within a subgroup increases. Therefore, spreading out the samples within a subgroup over time will increase $\bar{R}$ and increases the estimate of σ, widen the control limits and will thus appear to decrease the process capability index. Conversely, it is possible to increase process capability by consecutive piece sampling, giving a small $\bar{R}$ and σ estimate.

11.5 Preliminary data collection

After having decided upon the quality characteristic which is to be controlled and the frequency and size of the subgroup, some initial inspection data or measurements have to be collected and analysed for the purpose of providing preliminary control chart values that are needed in determining the centre line and control limits to be drawn on the chart. The preliminary data may be collected subgroup by subgroup until the recommended 25 subgroups have been obtained from a continuous run of the production process. Care shall be exercised that, during the course of this initial data collection, the process is not unduly influenced intermittently by extraneous factors such as change in the feed of raw material, operators, operations, machine settings, etc. In other words, the process should exhibit a state of stability during the period when preliminary data are being gathered.

11.6 Out of control action plan

11.6.1 There is an important connection between the two types of variation found and the types of action necessary to reduce them. Control charts can detect presence of special causes of variation. Investigating and discovering the source of the special cause(s) and taking the remedial actions is usually the responsibility of operators, supervisors or engineers directly associated with the process. The Management is responsible for more than 80 % of the causes and must take action on the vital causes in the system. Special causes are identified locally and can usually be actioned by the process owners. Processes are often adjusted as remedial action when management action on the system is needed on the root cause which might be different sources of raw material, machine requirements and maintenance, gauging, skilled manpower, resources, or an unreliable method. Close teamwork is the key to long term continual improvement.

11.6.2 If the process is inherently non-capable or is capable but goes out of statistical control and is found to be producing nonconforming product, then 100 % inspection is normally instituted until the corrective actions are taken to make the process as capable.

11.6.3 Consistency of inspection needs to be assured. Measurement uncertainty needs to be kept in harmless tolerable limits.

12 Steps in the construction of control charts

12.1 Typical format of a standard control chart form

The steps involved in the construction of the $\bar{X}$ chart and the R chart, for the case when no standard values are given, are described in 12.2 to 12.4. They are described in the form of an example in Annex A. In the construction of other control charts, the same basic steps shall be followed but the computational method for determining control limits and centre line are different. A typical format of a standard control chart form is shown in Figure 5. Modifications to this form can be made with the particular requirements of a process control situation.

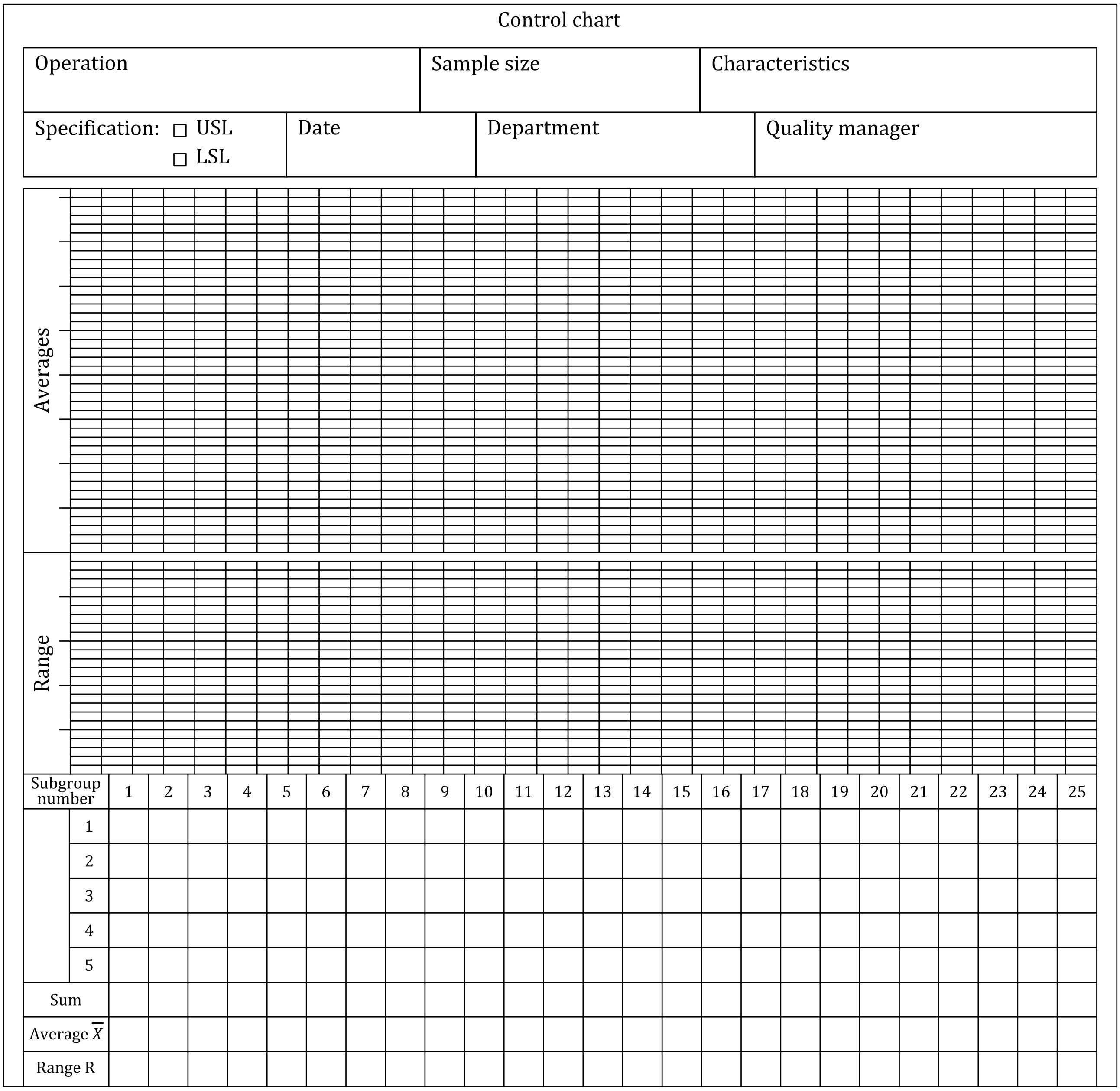


Figure 5 — General format of a variables control chart

12.2 Determine data collection strategy

If the preliminary data are not taken in subgroups according to a prescribed plan, break up the total set of observed values into sequential subgroups, according to the criteria for rational subgroups as discussed in 11.3. The subgroups must be of the same structure and size. The items of any one subgroup should have what is believed to be some important common factor, for example items produced during the same short interval of time or items coming from one of several distinct sources or locations. The different subgroups should represent possible or suspect differences in the process that produced them, for example different intervals of time or different sources or locations. A systems approach to the construction of variables control charts is given in Figure 6.

NOTE Prepare a list of known sources of chance and assignable causes of variation.

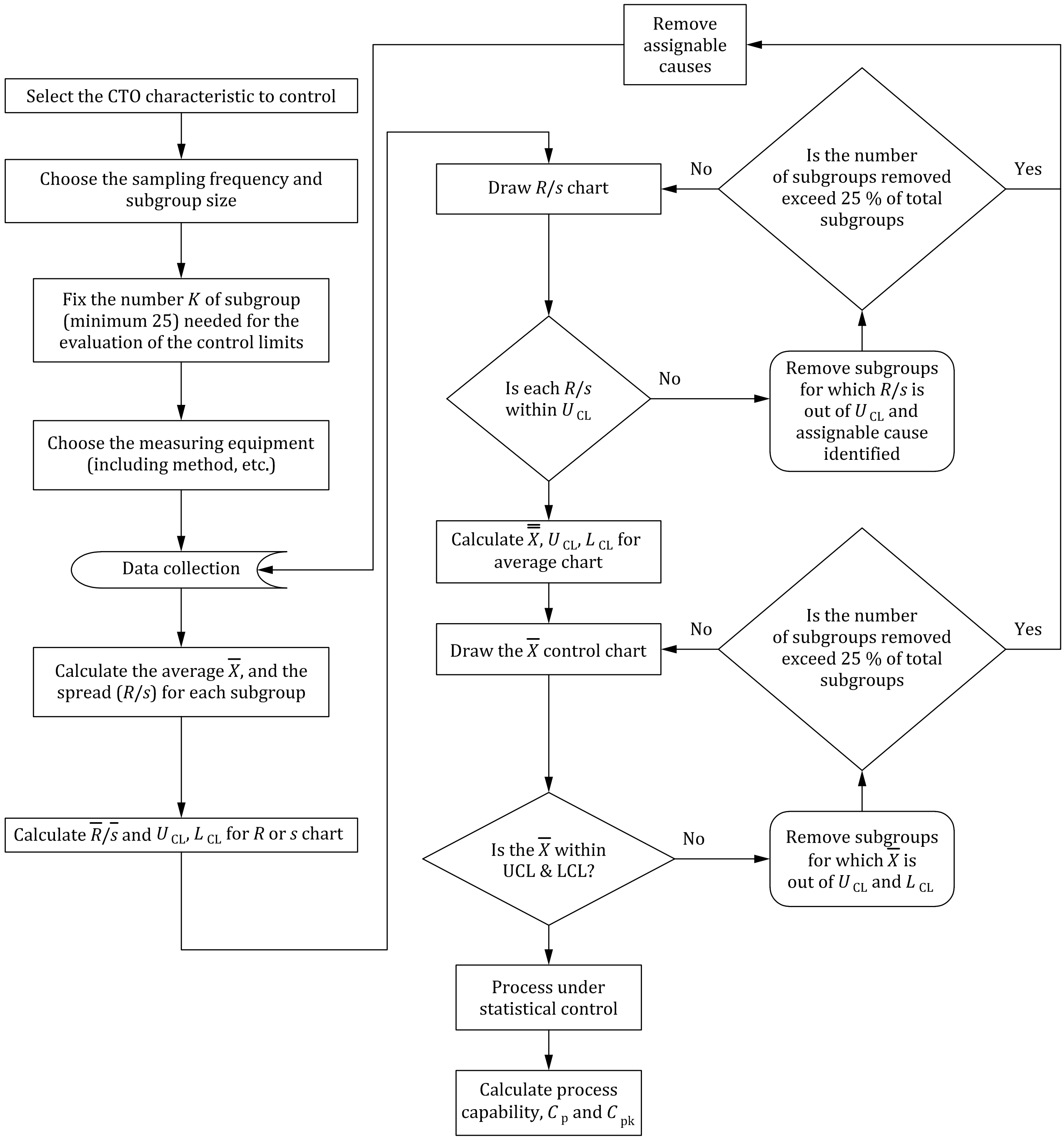


Figure 6 — Systems approach to the construction of variables control charts

12.3 Data collection and computation

For each subgroup, calculate the average, $\overline{X}$, and the range, R. Then, compute the grand average of all the observed values, $\overline{\overline{X}}$, and the average range, $\overline{R}$.

12.4 Plotting $\overline{X}$ chart and R chart

12.4.1 On a suitable form or graph paper, layout $\overline{X}$ chart and R chart. The vertical scale on the left is used for $\overline{X}$ and for R and the horizontal scale is used for the subgroup number. Plot the computed values for $\overline{X}$ on the chart for averages and plot the computed values for R on the chart for ranges. On the respective charts, draw solid horizontal lines to represent $\overline{\overline{X}}$ and $\overline{R}$.

12.4.2 Place the control limits on these charts. On the $\overline{X}$ chart, draw two horizontal dashed lines at $\overline{\overline{X}} \pm A_2 \overline{R}$, and on the R chart, draw two horizontal dashed lines at $D_3 \overline{R}$ and $D_4 \overline{R}$, where A_2, D_3 and D_4 are based on n, the number of observations in a subgroup, the values of which are given in Table 2. The L_{CL} on the R chart is not needed whenever n is less than 7 since the value of D_3 is zero.

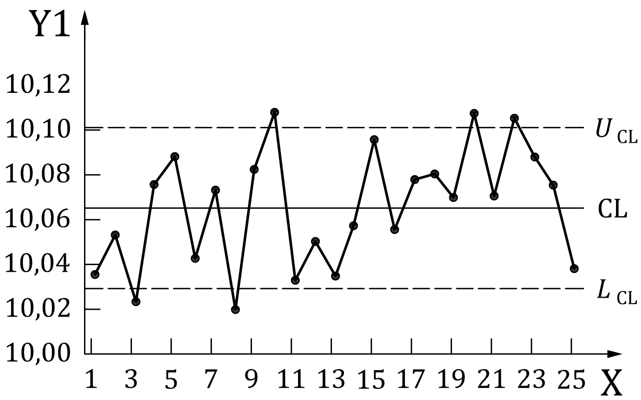
13 Caution with Shewhart control charts

13.1 General caution

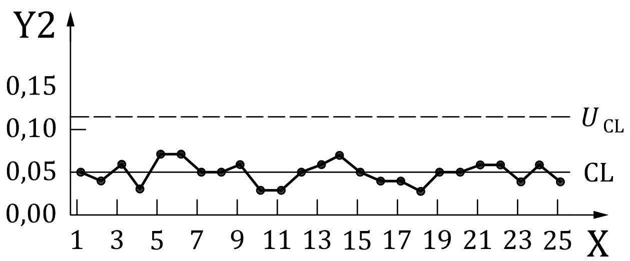
13.1.1 There are some practical situations, as given below, where some caution may be needed in using Shewhart control chart.

13.1.2 Sometimes it may be difficult to understand the variation due to chance causes by using the variation within a subgroup alone. The variation within a subgroup may not necessarily be due to chance causes alone. It may be difficult to identify some assignable causes and therefore variations due to such assignable causes has also to be included (see 7.3, NOTE 2). This means that the random variability due to some allowable causes between subgroups is regarded as the variability due to chance causes. For example, if the subgroup is composed of a lot, then the variability within a subgroup is the variability within a lot. The subgroup has a meaning from the viewpoints of both physical aspect and quality assurance. Therefore, it is necessary to control the variability within a lot. The following is an example of such a case.

Figure 7 shows $\overline{X}$ and R control chart in the early-stage mass production of a heat treatment process. This is $\overline{X}$ and R control chart where no standard values are given. R chart indicates process in state of control, but $\overline{X}$ chart shows many points and situations out-of-control.



a) $\overline{X}$ chart



b) R chart

Key

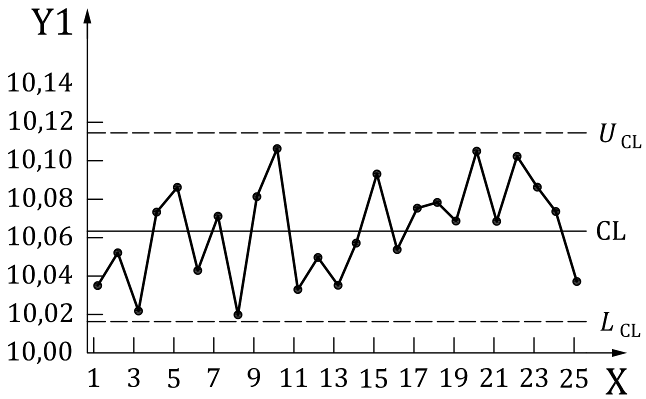
X subgroup number
Y1 average
Y2 range
CL for a) centre line = average of subgroup averages
for b) centre line range average
L_{CL} lower control limit
U_{CL} upper control limit

Figure 7 — Ordinary $\bar{X}$ and R chart in the early-stage mass production

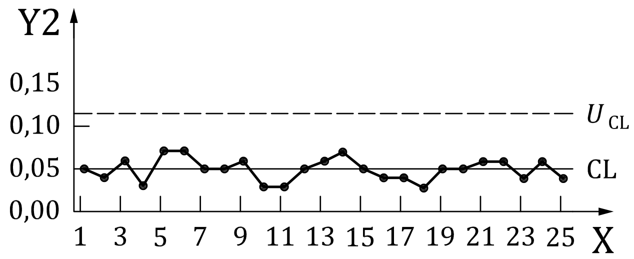
13.1.4 On the other hand, Figure 8 shows another $\bar{X}$ and R chart for the same data as that in Figure 7, where the control limits of $\bar{X}$ chart are calculated on the basis of the variability of $\bar{X}$ instead of the average of ranges, $\bar{R}$.

Figure 8 indicates that process is in-control. At that time if the process performance is well satisfied, it can be decided that the process can proceed to the routine mass production stage from the early-stage mass production. Then the control limits of $\bar{X}$ and R control chart in Figure 8 are used as a standard control level in the routine mass production. This means that the random variability due to some allowable causes between subgroups in the early-stage mass production is included as the variability due to chance causes.

13.1.6 Therefore, it should be noted that variability within a subgroup does not necessarily mean variability due to chance causes only. However, 17 to 24 points on $\bar{X}$ chart falling above the centre line, and the increasing trend from 9 to 24 points, along with clustering of points about $\bar{R}$ on range chart, do indicate potential for improvement through detection and elimination of assignable causes.



a) $\bar{X}$ chart



b) ***R*** **chart**

Key

X	subgroup number
Y1	average
Y2	range
CL	for a) centre line = average of subgroup averages = 10,065 3 for b) centre line = range average = 0,049 6
L_{CL}	lower control limit = 10,016 1
U_{CL}	for a) upper control limits =10,114 5 for b) upper control limits = 0,113 2

Figure 8 — $\bar{X}$ **and** ***R*** **chart**

13.2 Correlated data

13.2.1 In the presence of data correlation, the following equation, which is a fundamental equation in conducting a $\bar{X}$ chart with the sample size n, does not hold:

$$\sigma^2(\bar{X}) = \sigma^2 \text{ (individuals)}/n$$

13.3 Use of alternative rules to the three-σ rule

13.3.1 Shewhart control chart for the average will detect a large sustained shift in the process average quickly. However, if the shift in the average is small, of magnitude 1,5 standard deviation or less, Shewhart $\bar{X}$ control chart does not perform well. Therefore, in such cases, if the small shift in the process average from a desirable level has to be detected quickly, then additional pattern tests are usually employed. However, such supplemental rules may increase the false alarm rate, that is, the probability of observing a signal on the chart through the application of these rules increases substantially. On the other hand, when the control chart without standard values is used in the early-stage mass production, the supplementary rules given in Clause 8 should be considered for improving process performance. Alternative strategy is to use the control charts, such as in ISO 7870-4[1] or ISO 7870-6[2].

Annex A
(informative)

Illustrative examples

A.1 Variables control charts

A.1.1 $\overline{X}$ chart and R chart – μ and σ unknown

A.1.1.1 A supplier of houses for water pumps wishes to control a turning process using a control chart. An important characteristic is the bearing diameter. Measurements (in millimetres up to 3 places of decimal) are taken on consecutive 5 pumps every hour (subgroup size 5) for total 25 subgroups. The averages and ranges of each subgroup are given in Table A.1.

Table A.1 — Subgroup results from measurement of bearing diameter

j	$\overline{X}_j$	R_j
1	14,076 4	0,010
2	14,072 6	0,012
3	14,075 4	0,008
4	14,077 0	0,007
5	14,070 8	0,025
6	14,069 8	0,025
7	14,077 0	0,009
8	14,074 4	0,025
9	14,070 4	0,009
10	14,074 4	0,022
11	14,076 6	0,009
12	14,056 8	0,011
13	14,076 8	0,023
14	14,069 2	0,012
15	14,071 6	0,019
16	14,074 8	0,021
17	14,075 4	0,017
18	14,073 4	0,017
19	14,074 8	0,035
20	14,075 4	0,033
21	14,073 2	0,017
22	14,074 0	0,025
23	14,070 8	0,017
24	14,076 0	0,017
25	14,072 2	0,018
TOTAL	351,829 2	0,443

A.1.1.2 The first step is to plot an R chart (see Figure A.1) and evaluate its state of control. The values of D_3 and D_4 are taken from Table 2 where n = 5.

***R* chart:**

$$C_L = \overline{R} = \frac{1}{25}\sum_{j=1}^{25} R_j = \frac{0,443}{25} = 0,017\,72 = 0,017\,7 \text{ mm}$$

$$U_{\text{CL}} = D_4 \times \overline{R} = 2,114 \times 0,017\,72 = 0,037\,46 = 0,037\,5 \text{ mm}$$

$$L_{\text{CL}} = D_3 \times \overline{R}\text{, where } D_3 = 0 \text{ mm}$$

Since all range values are below upper control limit, the R chart indicates a process in control.

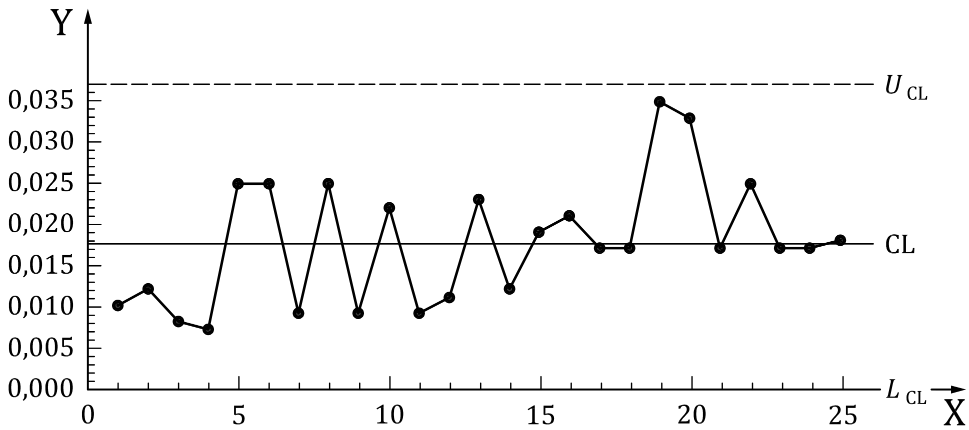
$\overline{X}$ chart:

$$\text{Centre line CL} = \overline{\overline{X}} = \frac{1}{25}\sum_{j=1}^{25} \overline{X}_j = 14,073\,17 \text{ mm}$$

$$U_{\text{CL}} = \overline{\overline{X}} + \text{A2} \times \overline{R} = 14,073\,17 + (0,577 \text{ x } 0,017\,72) = 14,083\,4 \text{ mm}$$

$$L_{\text{CL}} = \overline{\overline{X}} - \text{A2} \times \overline{R} = 14,073\,17 - (0,577 \text{ x } 0,017\,72) = 14,062\,9 \text{ mm}$$

The value of the factor A_2 is taken from Table 2, where n = 5.



Key

X subgroup number

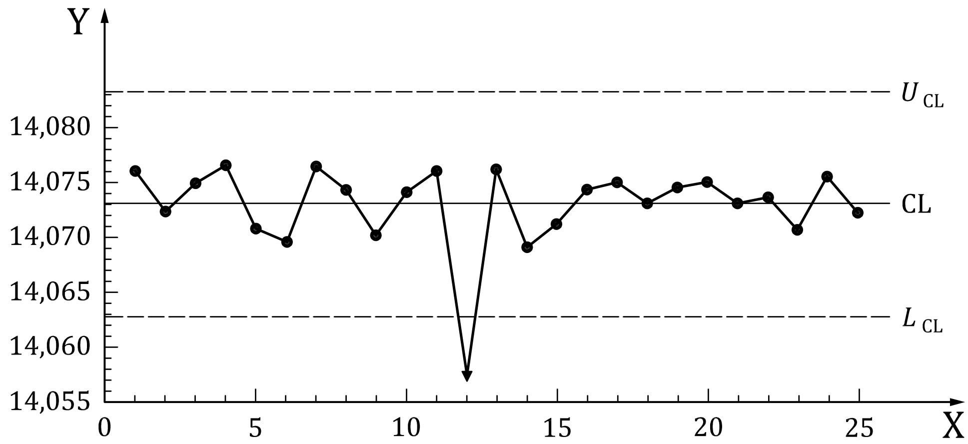
Y range

CL centre line = average range

L_{CL} lower control limit

U_{CL} upper control limit

Figure A.1 — *R* chart



Key

X subgroup number
Y average
CL centre line = average of subgroup averages
L_{CL} lower control limit
U_{CL} upper control limit

Figure A.2 — $\overline{X}$ chart

A.1.1.3 The examination of the $\overline{X}$ chart (see Figure A.2) reveals that subgroup 12 is below the lower control limit. It indicates that some assignable causes of variation may be operating. The subgroup 12 is discarded. For remaining 24 subgroups,

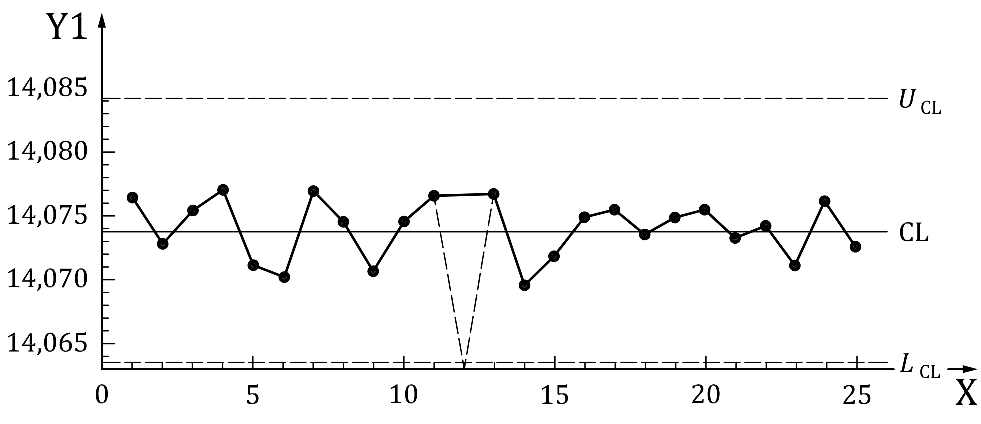
$$\overline{\overline{X}} = \frac{1}{24}\sum_{j=1}^{24}\overline{x}_j = \frac{337{,}7724}{24} = 14{,}07385$$

The revised $\overline{X}$ chart

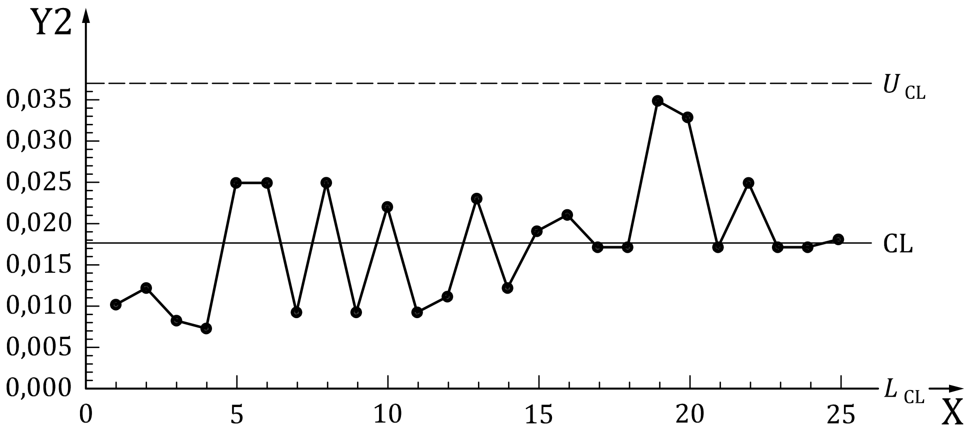
CL = $\overline{\overline{X}}$ = 14,073 8 mm

U_{CL} = $\overline{\overline{X}} + A_2 \times \overline{R}$ = 14,073 85 + (0,577 × 0,017 72) = 14,084 1 mm

L_{CL} = $\overline{\overline{X}} - A_2 \times \overline{R}$ = 14,073 85 − (0,577 × 0,017 72) = 14,063 6 mm



a) $\overline{X}$ chart



b) *R* chart

Key

X subgroup number

Y1 average

Y2 range

CL for a) centre line= average of subgroup averages
for b) centre line= average range

L_{CL} lower control limit

U_{CL} upper control limit

Figure A.3 — $\overline{X}$ and *R* chart

A.1.1.4 Now all the average values are within control limits. The above calculated control limits should be used to control the process in the future (see Figure A.3).

NOTE The reported values of control limits should be one more place of decimal than the original data. Like in this example, the original set of observations are up to 3 places of decimal (as range values given in table are up to 3 places of decimal). Therefore, the control limits for average and range charts are up to four places of decimal.

A.1.2 $\overline{X}$ chart and *s* chart – μ and σ known

A.1.2.1 A producer of batteries wishes to control the mass of his batteries such that the average mass of the batteries is 29,87 g. A process analysis from a former production has shown that the standard deviation of the process is 0,062 g. For this purpose, he collects 5 batteries everyday (subgroup size 5) for 25 days (number of subgroups is 25). The subgroup average and standard deviation are calculated, as shown in Table A.2.

A.1.2.2 Since the standard values are μ_0 = 29,87 g and σ_0 = 0,062 g, the control chart limits are calculated below using the formulae given in Table 1 and the factors A, C_4, D_2 and D_1 given in Table 2 using a subgroup size of 5.

Control limits for *s*-chart

$$\text{CL} = C_4\sigma_0 = 0{,}94 \times 0{,}062 = 0{,}058\,28 \text{ g} \approx 0{,}058\,3 \text{ g}$$

$$U_{\text{CL}} = B_6\sigma_0 = 1{,}964 \times 0{,}062 = 0{,}121\,768 \text{ g} \approx 0{,}121\,8 \text{ g}$$

$L_{CL} = B_5\sigma_0 = 0$, as $B_5 = 0$

Control limits for $\bar{X}$ -chart

$CL = \mu_0 = 29{,}87$ g

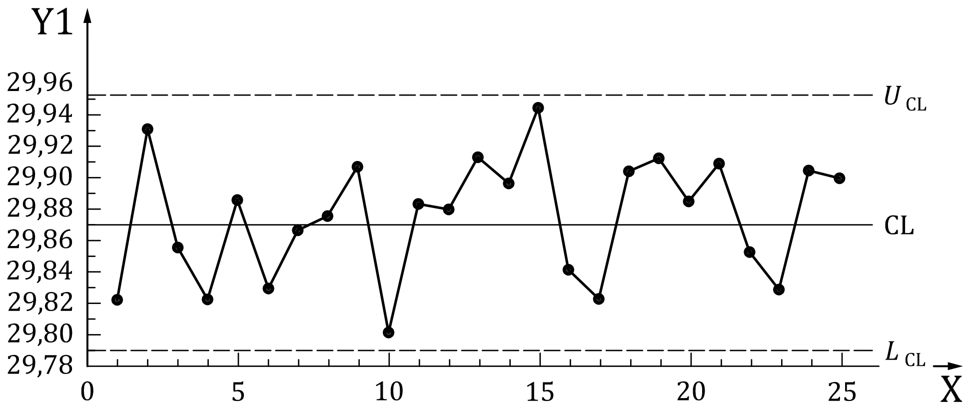
$U_{CL} = \mu_0 + A\sigma_0 = 29{,}87 + (1{,}342 \times 0{,}062) = 29{,}953\,2$ g

$L_{CL} = \mu_0 - A\sigma_0 = 29{,}87 - (1{,}342 \times 0{,}062) = 29{,}786\,8$ g

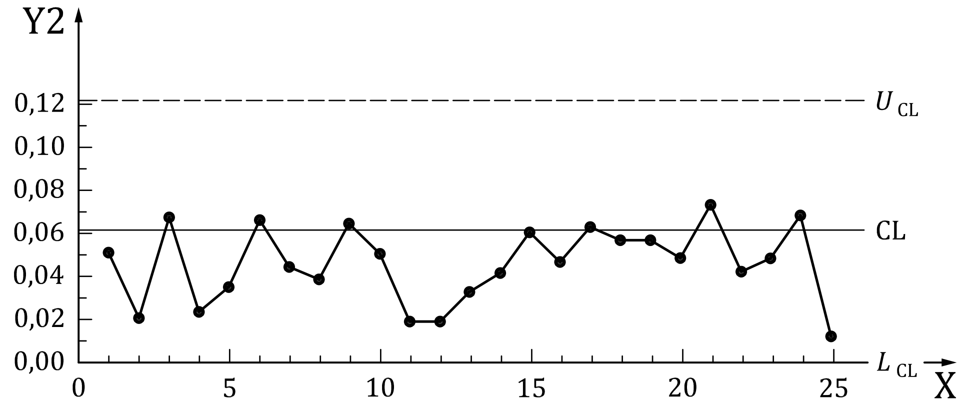
Table A.2 — Average and standard deviation of mass of batteries from production

j	$\bar{X}_j$	s_j
1	29,816	0,052
2	29,932	0,022
3	29,858	0,066
4	29,824	0,023
5	29,888	0,036
6	29,830	0,066
7	29,868	0,043
8	29,876	0,038
9	29,910	0,064
10	29,802	0,049
11	29,884	0,019
12	29,880	0,019
13	29,916	0,031
14	29,898	0,040
15	29,946	0,058
16	29,842	0,045
17	29,824	0,063
18	29,904	0,056
19	29,912	0,056
20	29,886	0,048
21	29,908	0,073
22	29,852	0,041
23	29,828	0,048
24	29,904	0,065
25	29,902	0,013

The subgroup results are plotted together with the control limits (see Figure A.4).



a) $\overline{X}$ chart



b) s chart

Key

X	subgroup number
Y1	average
Y2	standard deviation
CL	for a) centre line= average of subgroup averages for b) centre line= average standard deviation
L_{CL}	lower control limit
U_{CL}	upper control limit

Figure A.4 — $\overline{X}$ chart and s chart

A.1.2.3 The control chart shown in Figure A.4 indicates that the process is in statistical control.

A.1.3 Control charts for individuals and moving ranges - μ and σ unknown

A.1.3.1 General

A sample of skim milk powder, representing a lot, is analysed in the laboratory for moisture content. It is intended to control the moisture below 4 %. The sampling variation within a lot is found to be negligible, so it is decided to take only one sample per lot. The laboratory analysis results of moisture content from 25 successive lots are given in Table A.3. Calculate control limits for individual and moving range control charts.

Table A.3 — Percent moisture for 25 successive samples of skim milk powder

Lot No.	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**
X: % moisture	2,9	3,2	3,6	4,3	3,8	3,5	3,0	3,1	3,6	3,5	3,1	3,4	3,4
R_m		0,3	0,4	0,7	0,5	0,3	0,5	0,1	0,5	0,1	0,4	0,3	0
Lot No.	**14**	**15**	**16**	**17**	**18**	**19**	**20**	**21**	**22**	**23**	**24**	**25**	
X: % moisture	3,6	3,3	3,9	3,5	3,6	3,3	3,0	3,4	3,8	3,5	3,2	3,5	
R_m	0,2	0,3	0,6	0,4	0,1	0,3	0,3	0,4	0,4	0,3	0,3	0,3	

A.1.3.2 Control limits for moving ranges R_m

$$\text{CL} = \overline{R_\text{m}} = 8/24 = 0{,}333 \approx 0{,}33$$

$$U_\text{CL} = D_4 \overline{R_\text{m}} = 3{,}267 \times 0{,}333 = 1{,}087\,9 \approx 1{,}09$$

$$L_\text{CL} = D_3 \overline{R_\text{m}} = 0 \times 0{,}333 = 0$$

The formulae for control limits are given in Table 3. The values of D_3 and D_4 are given in Table 2 for n = 2. Since all values of ranges are below upper control limit, the range chart exhibits a state of statistical control, and control limits for individual control chart can be carried out.

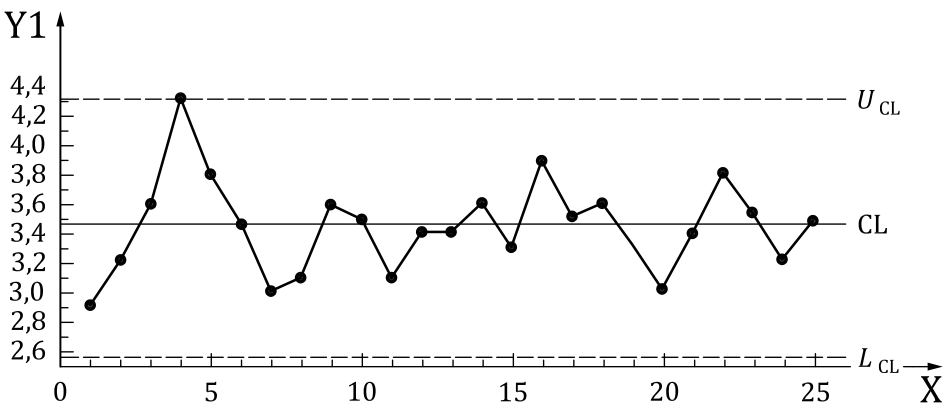
A.1.3.3 Control chart for individuals, X

$$\text{CL} = \bar{X} = 86/25 = 3{,}440$$

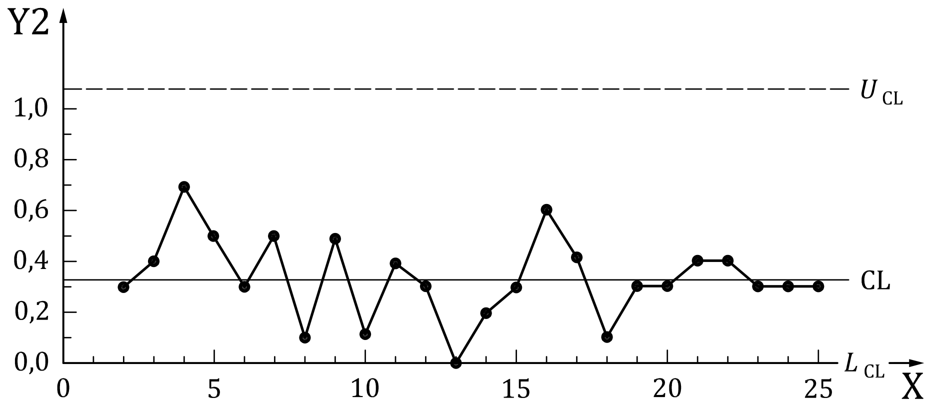
$$U_\text{CL} = \bar{X} + 2{,}660\ \overline{R_\text{m}} = 3{,}44 + (2{,}66 \times 0{,}33) = 4{,}326$$

$$L_\text{CL} = \bar{X} - 2{,}660 A_2\ \overline{R_\text{m}} = 3{,}44 - (2{,}66 \times 0{,}33) = 2{,}554$$

Since all individual values of moisture content are within control limits, the individual values of all subgroups exhibit a state of statistical control. The control charts are plotted in Figure A.5 shows that the process is in statistical control.



a) Individual values X chart



b) Moving range R_m chart

Key

X subgroup number

Y1 individuals

Y2 moving range

CL for a) centre line= average of individuals
for b) centre line= average of moving range

L_{CL} lower control limit

U_{CL} upper control limit

Figure A.5 — Control chart for individuals, X and moving range, R_m

A.1.4 Median and R chart - μ and σ unknown

A.1.4.1 General

An ordinary Portland cement is packed in bags of nominal mass of 50 kg. The "mass" of a bag is important characteristic and requires to be controlled. For this purpose, a sample of 50 kg bag from each of the 5 nozzles of a machine is taken every hour, and checked for its mass. The results so obtained are given in Table A.4. The values of medians and ranges are also shown in Table A.4.

Table A.4 — Mass of cement bags

Subgroup No.	Mass kg					Median	Range
1	50,00	50,20	50,00	50,40	50,60	50,20	0,60
2	50,20	50,00	50,40	50,40	50,40	50,40	0,40
3	50,60	50,40	50,20	50,20	50,60	50,40	0,40
4	50,80	50,20	49,50	50,20	49,80	50,20	1,30
5	50,40	49,80	49,20	50,20	50,40	50,20	1,20
6	50,80	50,40	50,60	50,20	50,60	50,60	0,60
7	50,20	50,20	50,80	50,80	50,20	50,20	0,60
8	50,60	50,60	50,40	50,20	50,80	50,60	0,60
9	50,80	50,10	50,20	50,30	49,60	50,20	1,20
10	50,20	50,40	50,20	49,80	50,00	50,20	0,60
11	50,40	50,20	50,60	50,00	50,20	50,20	0,60
12	50,40	50,40	49,80	50,20	50,40	50,20	0,60

Table A.4 *(continued)*

Subgroup No.	Mass kg					Median	Range
13	50,60	50,20	50,40	50,80	50,20	50,40	0,60
14	50,60	49,80	49,20	50,40	50,80	50,40	0,60
15	49,50	50,20	50,20	50,50	50,80	50,40	0,60
16	50,60	50,20	50,40	49,80	50,40	50,60	1,60
17	50,80	50,20	51,00	50,40	50,20	50,20	1,30
18	52,10	52,30	52,50	51,50	51,50	52,10	1,00
19	51,00	52,30	52,10	52,50	52,50	52,30	1,50
20	50,40	52,40	52,30	52,40	51,50	52,30	2,00
21	50,80	50,20	50,80	50,40	50,00	50,40	0,80
22	50,80	50,40	50,20	50,60	50,50	50,50	0,60
23	50,50	50,00	50,80	50,60	50,80	50,60	0,80
24	50,40	50,40	49,80	50,20	50,40	50,40	0,60
25	50,40	50,40	49,80	5,20	50,40	50,40	1,00
		Total				1 264,60	21,70

A.1.4.2 Control limits for range chart

$$\text{CL} = \overline{R} = 21{,}70/25 = 0{,}868$$

$$U_{\text{CL}} = D_4\overline{R} = 2{,}115 \times 0{,}868 = 1{,}836$$

As range for subgroup number 20 is greater than U_{CL}, this range value is deleted for homogenization and calculations are done again.

$$\text{CL} = \overline{R} = (21{,}7 - 2{,}0)/24 = 0{,}821$$

$$U_{\text{CL}} = 2{,}115 \times 0{,}821 = 1{,}736$$

$$L_{\text{CL}} = 0 \times 0{,}821 = 0$$

Since all the values of range are within U_{CL}, the above revised values are the control limits for range chart.

A.1.4.3 Control limits for median chart

Subgroup 20 has been discarded above while homogenising range values. Therefore, for remaining 24 subgroups.

$$\text{CL} = \overline{\tilde{X}} = (1\ 264{,}6 - 52{,}3)/24 = 1\ 212{,}3/24 = 50{,}513$$

$$U_{\text{CL}} = \overline{\tilde{X}} + A_4\ \overline{R} = 50{,}513 + 0{,}691 \times 0{,}821 = 50{,}513 + 0{,}567 = 51{,}080$$

$$L_{\text{CL}} = \overline{\tilde{X}} - A_4\ \overline{R} = 50{,}513 - 0{,}567 = 49{,}946$$

The value for A_4 is given in Table 4.

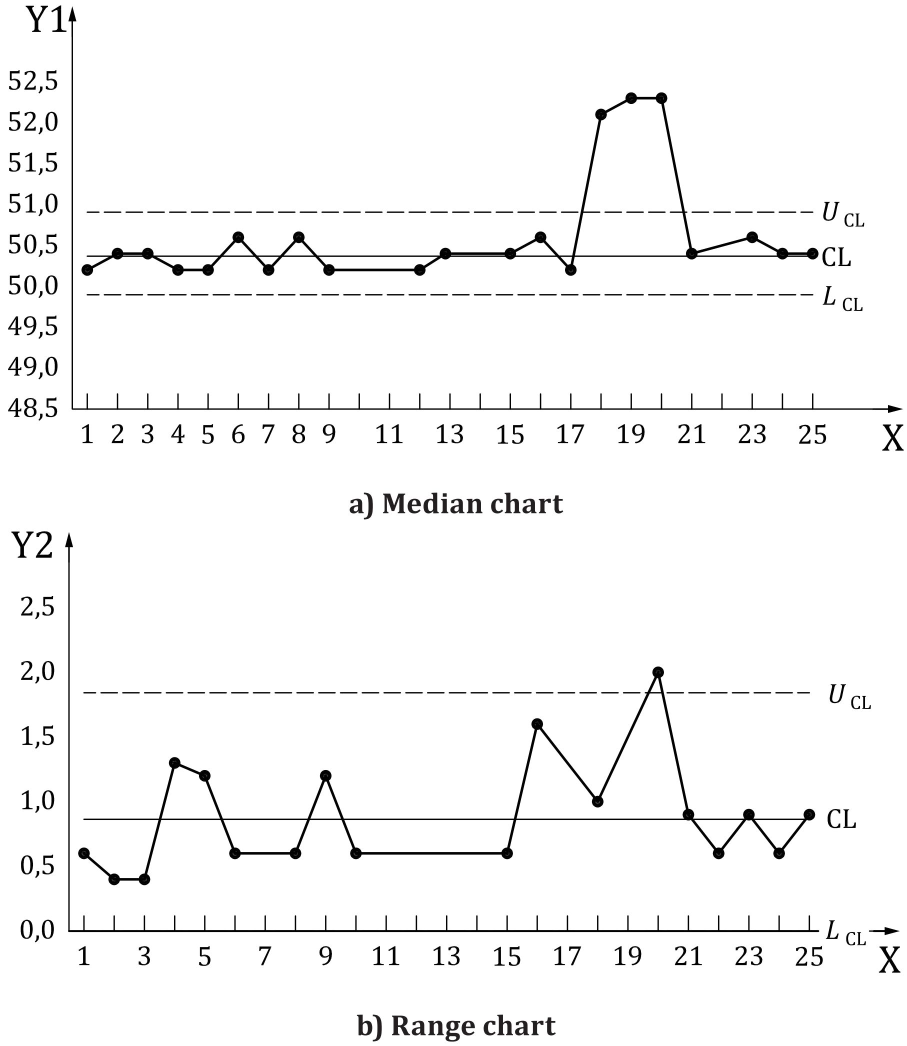
Since the median values for subgroup number 18 and 19 are more than U_{CL}, these values are deleted for homogenization. For remaining 22 subgroups:

$$CL = \bar{\bar{X}} = (1\,212{,}3 - 52{,}1 - 52{,}3)/22 = 1\,107{,}9/22 = 50{,}359$$

$$U_{CL} = 50{,}359 + 0{,}538 = 50{,}897$$

$$L_{CL} = 50{,}359 - 0{,}538 = 49{,}821$$

Since all the median values are within L_{CL} and U_{CL}, these are taken as the control limits for median chart.



Key

X	hour
Y1	median
Y2	range
CL	for a) centre line= average of medians for b) centre line= average range
L_{CL}	lower control limit
U_{CL}	upper control limit

Figure A.6 — Median and range control chart

A.2 Attribute control charts

A.2.1 *p* chart- no p_0 value given

A.2.1.1 In a manufacturing company producing radio transistors, it is decided to install a fraction nonconforming *p* chart. Data are collected and analysed for a period of one month. From each day's production a random sample is collected at the end of the day and examined for the number of nonconforming items. The data are shown in Table A.5.

Table A.5 — Radio transistors: *p* chart (initial data)

Day	Number inspected	Number nonconforming	Fraction nonconforming
1	158	11	0,070
2	140	11	0,079
3	140	8	0,057
4	155	6	0,039
5	160	4	0,025
6	144	7	0,049
7	139	10	0,072
8	151	11	0,073
9	163	9	0,055
10	148	5	0,034
11	150	2	0,013
12	153	7	0,046
13	149	7	0,047
14	145	8	0,055
15	160	6	0,038
16	165	15	0,091
17	136	18	0,132
18	153	10	0,065
19	150	9	0,060
20	148	5	0,034
21	135	0	0,000
22	165	12	0,073
23	143	10	0,070
24	138	8	0,058
25	144	14	0,097
26	161	20	0,124
Total	3 893	233	

A.2.1.2 The values of the fraction nonconforming calculated for each subgroup are also given in Table A.5. The average fraction nonconforming for the month is calculated as follows:

$$\overline{p} = \frac{233}{3\,893} = 0{,}06$$

Since subgroup sizes are different, the U_{CL} and L_{CL} values shall be calculated for each subgroup separately from:

$$U_{CLi} = \overline{p} + 3\sqrt{\frac{\overline{p}(1-\overline{p})}{ni}}$$

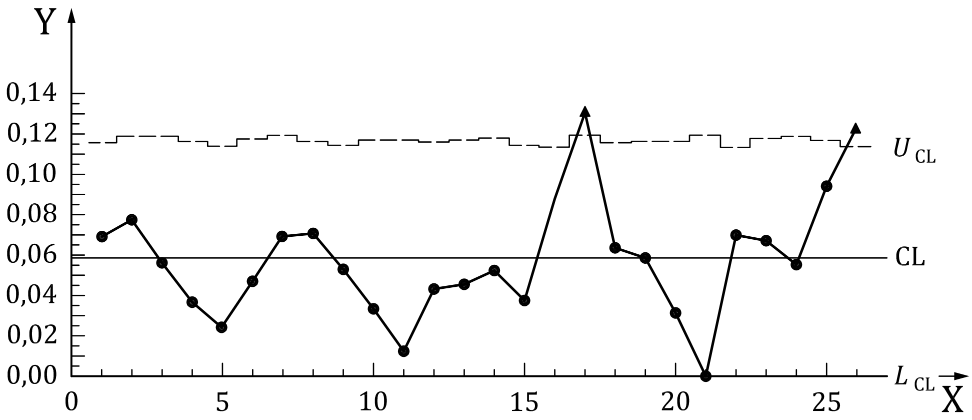
$$L_{CLi} = \overline{p} - 3\sqrt{\frac{\overline{p}(1-\overline{p})}{ni}}$$

where n_i is the size of the i^{th} subgroup.

Table A.6 shows the calculation results for every subgroup.

Table A.6 — Radio transistors – calculation results

Subgroup number	Number inspected	Fraction nonconforming, p	U_{CL}	L_{CL}
1	158	0,070	0,117	0,003
2	140	0,079	0,120	0,000
3	140	0,057	0,120	0,000
4	155	0,039	0,117	0,003
5	160	0,025	0,116	0,004
6	144	0,049	0,119	0,001
7	139	0,072	0,120	0,000
8	151	0,073	0,118	0,002
9	163	0,055	0,116	0,004
10	148	0,034	0,119	0,001
11	150	0,013	0,118	0,002
12	153	0,046	0,118	0,002
13	149	0,047	0,118	0,002
14	145	0,055	0,119	0,001
15	160	0,038	0,116	0,004
16	165	0,091	0,115	0,005
17	136	0,132	0,121	0,000
18	153	0,065	0,118	0,002
19	150	0,060	0,118	0,002
20	148	0,034	0,119	0,001
21	135	0,000	0,121	0,000
22	165	0,073	0,115	0,005
23	143	0,070	0,120	0,000
24	138	0,058	0,121	0,000
25	144	0,097	0,119	0,001
26	161	0,124	0,116	0,004
Total	3 893			



Key

X	subgroup number
Y	fraction nonconforming
CL	centre line
L_{CL}	lower control limit
U_{CL}	upper control limit

Figure A.7 — *p* chart

A.2.1.3 Plotting the U_{CL} and L_{CL} values for each subgroup is a time-consuming task. It can be observed from Table A.6 and Figure A.7 that the fractions nonconforming for subgroup numbers 17 and 26 are falling outside their corresponding upper control limits. These two subgroups are discarded for the purpose of homogenization. A revised average fraction nonconforming is calculated from the remaining 24 subgroup values:

$$\overline{p} = \frac{195}{3\,596} = 0,054$$

A.2.1.4 Calculating the revised U_{CL} and L_{CL} values for each subgroup, using the revised $\overline{p}$ value, would reveal that all the fractions nonconforming are within their corresponding control limits. Hence, this revised value of $\overline{p}$ is taken as the standard fraction nonconforming for the purpose of installation of control charts for future purpose (Phase 2). Thus, p_0 = 0,054.

A.2.1.5 As mentioned above, the plotting of upper control limits for each subgroup of varying sizes is a time consuming and tedious process. However, since the subgroup sizes do not vary widely from the average subgroup size, which comes out to be 150, the revised *p* chart (using p_0 = 0,054) can be plotted with an upper control limit using a subgroup size of n = 150, as the average subgroup size.

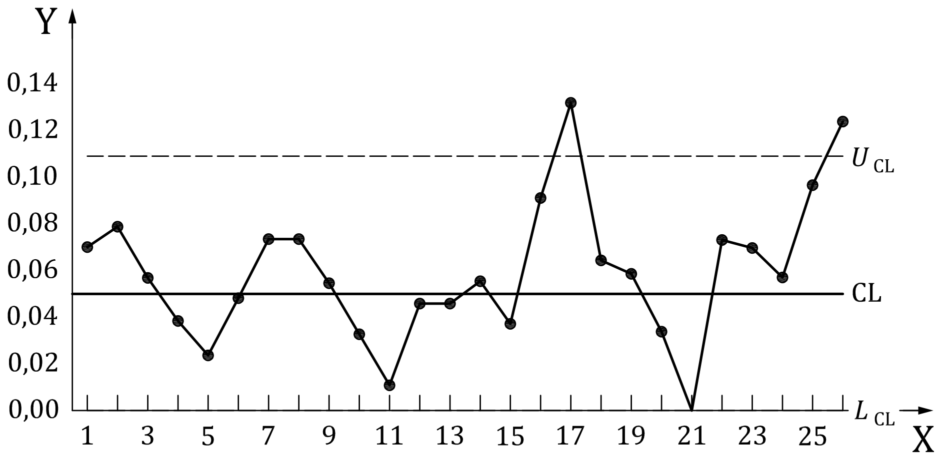
$$C_L = p_0 = 0,054$$

$$U_{CL} = p_0 + 3\sqrt{\frac{p_0(1-p_0)}{n}} = 0,054 + 3\sqrt{\frac{0,054(1-0,054)}{150}} = 0,109$$

$$L_{CL} = p_0 - 3\sqrt{\frac{p_0(1-p_0)}{n}} = 0,054 - 3\sqrt{\frac{0,054(1-0,054)}{150}} = -0,001 = 0$$

NOTE Since negative values are not possible, the lower limit is shown as zero.

The revised *p* chart is plotted below in Figure A.8. The process is exhibiting a state of statistical control.



Key

X subgroup number

Y fraction nonconforming

CL centre line

L_{CL} lower control limit

U_{CL} upper control limit

Figure A.8 — Revised *p* chart

A.2.2 *np* chart - p_0 not given

A.2.2.1 General

The data in Table A.7 gives the number of nonconforming items per hour regarding malfunctions found by 100 % inspection on small switches with automatic inspection devices. The switches are produced in an automatic assembly line. Since the malfunction is serious, the percent nonconforming is used to identify when the assembly line is out of control. A *np* chart is prepared by gathering data of 25 subgroups as the preliminary data since the number inspected is constant.

Table A.7 — Preliminary data: switches

Subgroup number	Number of switches inspected	Number of nonconforming switches
1	4 000	8
2	4 000	14
3	4 000	10
4	4 000	4
5	4 000	13
6	4 000	9
7	4 000	7
8	4 000	11
9	4 000	15
10	4 000	13
11	4 000	5
12	4 000	14
13	4 000	12

Table A.7 *(continued)*

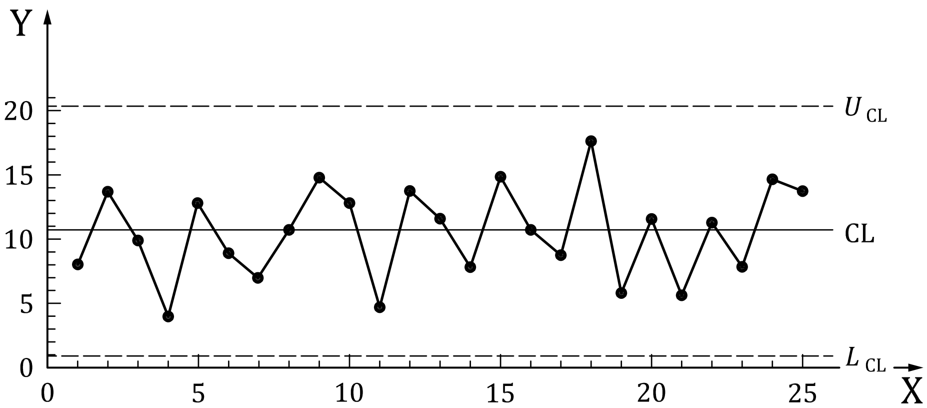
Subgroup number	Number of switches inspected	Number of nonconforming switches
14	4 000	8
15	4 000	15
16	4 000	11
17	4 000	9
18	4 000	18
19	4 000	6
20	4 000	12
21	4 000	6
22	4 000	12
23	4 000	8
24	4 000	15
25	4 000	14
Total	100 000	269

A.2.2.2 Control limits for *np* chart

$$\text{CL} = n\bar{p} = \frac{8+14+\ldots+14}{25} = 10{,}76$$

$$U_{\text{CL}} = n\bar{p} + 3\sqrt{n\bar{p}(1-\bar{p})} = 10{,}76 + 3\sqrt{10{,}76(1-0{,}0027)} = 20{,}59$$

$$L_{\text{CL}} = n\bar{p} - 3\sqrt{n\bar{p}(1-\bar{p})} = 10{,}76 - 3\sqrt{10{,}76(1-0{,}0027)} = 0{,}93$$



Key

X subgroup number
Y number nonconforming switches
CL centre line
L_{CL} lower control limit
U_{CL} upper control limits

Figure A.9 — *np* chart

A.2.2.3 All values of nonconforming items in Table A.7 are below upper control limit (see also Figure A.9), which indicate that the quality of switches is in statistical control. These control limits may now be used for future subgroups until such time that the process is altered or that the process goes out of statistical control. Since the process is in statistical control, it is unlikely that any improvement can be made without a process change. If an improvement is made, then different control limits will have to be computed for future subgroups to reflect the altered process performance. If the process has been improved (smaller *np* value), use the new limits, but if the process has deteriorated (higher *np* value), search for additional assignable causes.

A.2.3 *c* chart – c_0 not given

A.2.3.1 General

In a tyre manufacturing plant, 50 tyres are inspected every hour for visual nonconformities and the total number of nonconformities are recorded. Each subgroup consists of 50 tyres. It is decided to install *c* chart for the number of nonconformities to study the state of control of the process. The data are shown in Table A.8.

Table A.8 — Number of nonconformities in each subgroup consisting of 50 tyres

Subgroup no.	1	2	3	4	5	6	7	8	9	10
Number nonconformities	4	5	3	6	2	1	5	6	2	4
Subgroup no	11	12	13	14	15	16	17	18	19	20
Number nonconformities	7	5	2	3	5	1	2	6	3	5
Subgroup no	21	22	23	24	25	26	27	28	29	30
Number nonconformities	3	2	4	1	5	3	2	1	3	4

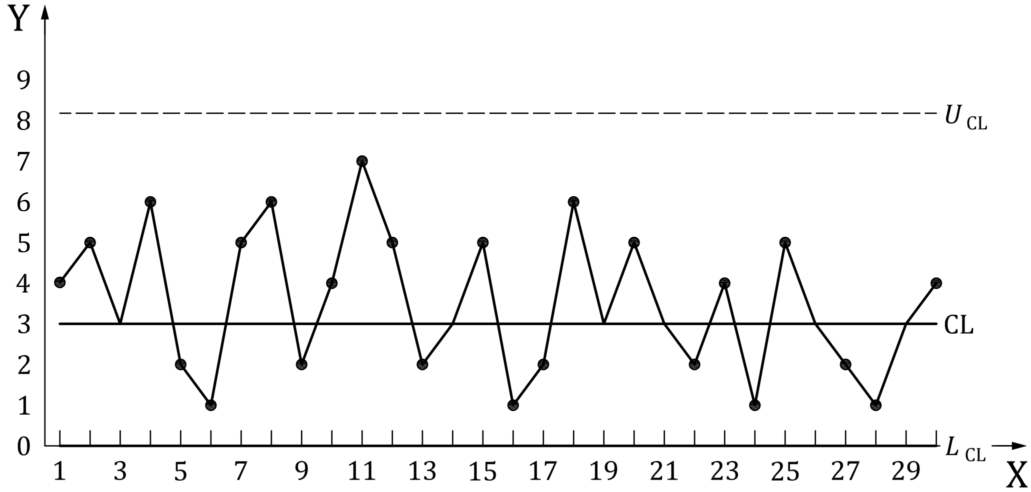
A.2.3.2 Control limits for *c* chart

$$\text{CL} = \bar{c} = 105/30 = 3{,}5$$

$$U_{\text{CL}} = \bar{c} + 3\sqrt{\bar{c}} = 3{,}5 + 3\sqrt{3{,}5} = 9{,}11$$

$$L_{\text{CL}} = \bar{c} - 3\sqrt{\bar{c}} = 3{,}5 - 3\sqrt{3{,}5} = 0$$

A.2.3.3 As values of number of nonconformities for all subgroups are below upper control limit, the above are taken as control limits, and the process is in statistical control, see Figure A.10.



Key

X	subgroup number
Y	number of nonconformities
CL	centre line = average of diameter averages
L_{CL}	lower control limit
U_{CL}	upper control limits

Figure A.10 — *c* chart

A.2.4 *u* chart - u_0 not given

A.2.4.1 General

In cast iron foundry, number of engine blocks produced in batch vary. It is decided that the entire batch should be taken as one subgroup. All the items from each batch are inspected and the number of nonconformities obtained are given in Table A.9. Since the number of items in each batch are varying, a control chart for number of nonconformities per item is installed. From the initial data, for each of the subgroups, the number of nonconformities per item are calculated and given in Table A.9.

A.2.4.2 Control limits for *u* chart

$$CL = \bar{u} = 153/476 = 0{,}32$$

$U_{CLi} = \bar{u} + 3\sqrt{\bar{u}/n_i}$, where n_i is the size of i^{th} subgroup

A.2.4.3 The U_{CL} corresponding to different subgroups are given in column 6 of Table A.9. It is observed that number of nonconformities per item corresponding to subgroup number 5, 12 and 14 are falling outside the corresponding U_{CL} and hence these three subgroups are discarded from the initial data. From the remaining 21 subgroups a revised average number of nonconformities per item is computed as follows:

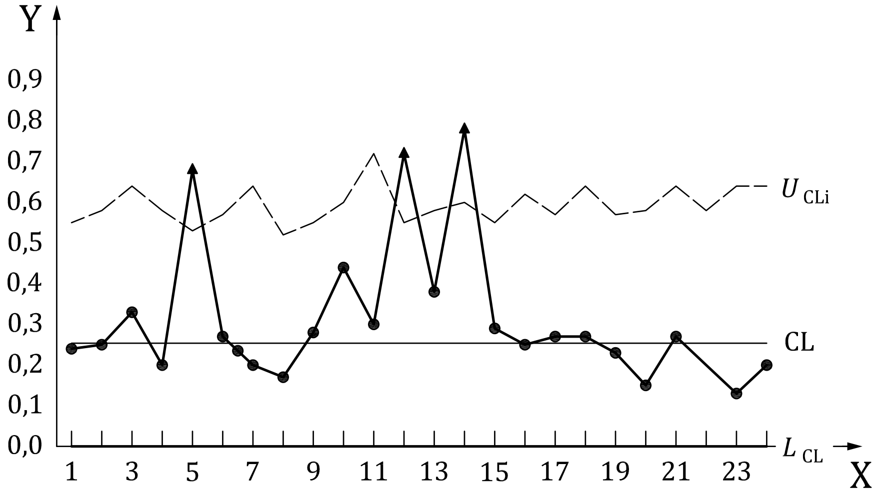
$$CL = (153 - 51)/(476 - 71) = 102/405 = 0{,}25$$

A.2.4.4 U_{CLi} for various subgroups are recalculated with the help of the revised average number of nonconformities per item, and are given in column 7 of Table A.9. It is observed that the number of nonconformities per item for each subgroup are now within respective U_{CL}. Hence, that revised value of the average number of nonconformities per item (equal to 0,25) is taken as the standard average number of nonconformities per item for the purpose of installation of control chart. The L_{CL} for each subgroup is coming out as negative, and hence taken as zero.

The control chart for number of nonconformities per item is given in Figure A.11.

Table A.9 — Number of nonconformities per item

Batch no.	Day	No. of items	No. of non-conformities	Non-conformities per Item	U_{CLi}	Revised U_{CLi}	Remarks
1	2	25	6	0,24	0,662	0,553	
2	2	20	5	0,25	0,703	0,589	
3	2	15	5	0,33	0,762	0,641	
4	3	20	4	0,20	0,703	0,589	
5	3	28	19	0,68	0,644	0,536	
6	3	22	6	0,27	0,685	0,573	
7	4	15	3	0,20	0,762	0,641	
8	4	30	5	0,17	0,633	0,527	
9	5	25	7	0,28	0,662	0,553	
10	5	18	8	0,44	0,723	0,607	
11	5	10	3	0,30	0,860	0,728	
12	6	25	18	0,72	0,662	0,553	
13	6	21	8	0,38	0,693	0,580	
14	6	18	14	0,78	0,723	0,607	
15	7	24	7	0,29	0,669	0,559	
16	8	16	4	0,25	0,748	0,628	
17	8	22	6	0,27	0,685	0,573	
18	8	15	4	0,27	0,762	0,641	
19	9	22	5	0,23	0,685	0,573	
20	9	20	3	0,15	0,703	0,589	
21	9	15	4	0,27	0,762	0,641	
22	10	20	4	0,20	0,703	0,589	
23	10	15	2	0,13	0,762	0,641	
24	10	15	3	0,20	0,762	0,641	
	Total	476	153				



Key

X	subgroup number
Y	number of nonconformities per item
CL	centre line = average of diameter averages
L_{CL}	lower control limit
U_{CL}	upper control limits

Figure A.11 — *u* chart

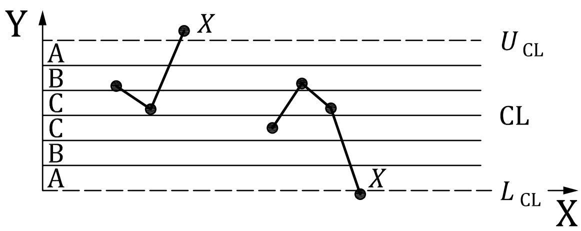
Annex B
(informative)

Practical notices on the pattern tests for assignable causes of variation

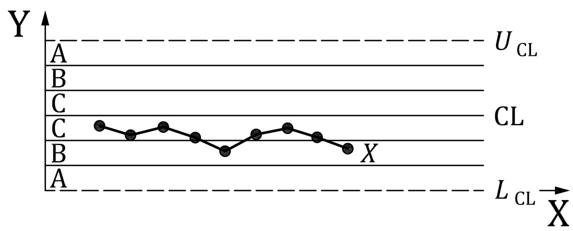
Practical notices on using the pattern tests in Figure 3 are given as follows

a) As shown in Clause 8, if some of the pattern tests in Figure 3 are used together, then the probability of Type 1 error may become too large. However, in early stage of production, the purpose of statistical process control is to bring the process into a stable state and improve the process for better process performance. Therefore, we must positively and rapidly detect presence of assignable causes by using pattern tests in Figure 3. On the other hand, when the production stage is transferred to the routine mass production, the purpose of statistical process control is to maintain the process in a state of control. In this case a very small probability of Type 1 error is required. Therefore, using some tests together should be avoided. Example 1 in Figure 3 is fundamental rule of Shewhart control chart.

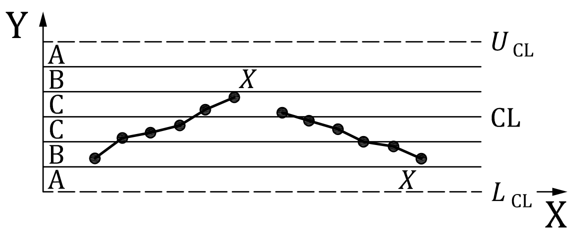
b) Western Electric Rules has also specified various supplementary rule giving different criteria for identifying assignable causes. Figure B.1 shows the eight typical test criteria given in these rules. For example, if a relatively small shift or a trend in the process average tends to appear, then it is helpful to use a supplementary rule. Test 5 in addition to Test 1 in Figure B.1. However, the decision as to which test (s) is to be used depends on the process being studied



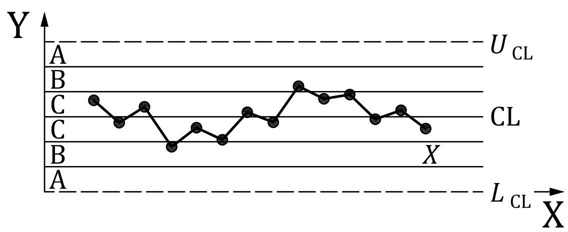
a) Test 1: One point beyond zone A



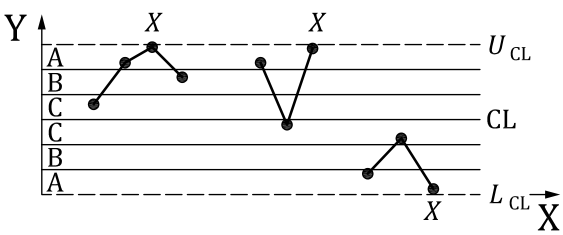
b) Test 2: Nine points in a row in zone C or beyond on one side of centre line

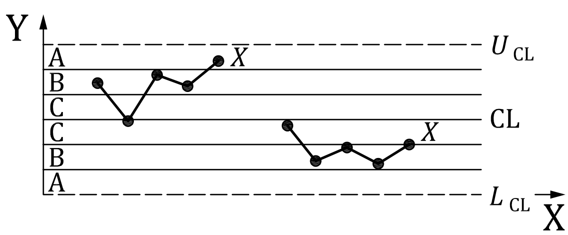


c) Test 3: Six points in a row vertically increasing or decreasing



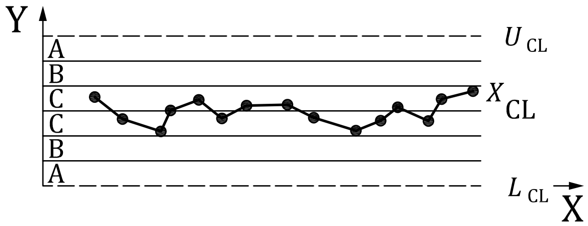
d) Test 4: Fourteen points in a row alternating up and down

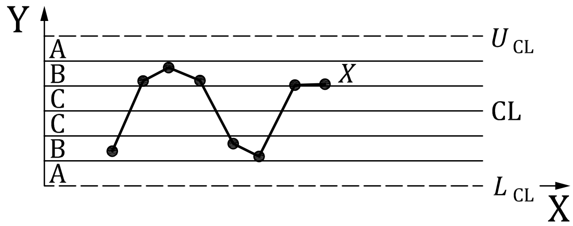




e) Test 5: Two out of three points in a row in zone A or beyond on one side of centre line

f) Test 6: Four out of five points in a row in zone B or beyond on one side of centre line





g) Test 7: Fifteen points in a row in zone C above and below centre line

h) Test 8: Eight points in a row on both sides of centre line with none in zone C

Key

X subgroup number

Y statistic

CL centre line

L_{CL} lower control limit

U_{CL} upper control limit

Figure B.1 — Tests for assignable causes

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