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(तीसरा पुनरीक्षण)

Lot Sampling — Manual on Basic Principles

(Third Revision)

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Statistical Methods for Quality, Data Analytics and Reliability Sectional Committee, MSD 03

FOREWORD

This Indian Standard (Third Revision) was adopted by the Bureau of Indian Standards, after the draft finalized by the Statistical Methods for Quality, Data Analytics and Reliability Sectional Committee had been approved by the Management and Systems Division Council.

Inspection of lots of products to get assurances about their quality, is a vital problem with the consumers. With phenomenal increase in the manufacturing capacity due to the machine age, inspection of each and every unit of production becomes extremely uneconomical. In many cases, like those involving destructive or costly testing, inspection of each and every unit is not to be conceived at all. The only alternative method available in these cases is sampling inspection, that is, selecting a smaller number of units called a sample from the lot or the production and then deciding about the quality of the latter on the basis of the results obtained from the inspection of the sample units. The sampling inspection is generally more practical, quick and economical.

The standard was first published in 1960 and revised in 1969 and 1981, with a view to acquaint the producers and consumers with the concepts and principles of sampling inspection.

In view of the experience gained with the use of the standard in course of years, it was felt necessary to comprehensively revise it again so as to make the concepts more up-to-date, harmonize various statistical terms with those of Indian Standards published subsequently, include terminology of new statistical terms used in the standard, revise existing material and include new material. The factual errors have also been corrected.

It is hoped that this standard will continue to contribute towards the increased use of sampling plans by consumers for judging the acceptability of their purchases and by the manufacturers for purposes of improving the quality of their products.

Besides, this standard is also expected to be of assistance to various Committees of Bureau of Indian Standards in familiarizing them with the concepts and principles of sampling and evolving sampling plans suited to individual specifications.

In this revision, refinements in statistical theory and advanced sampling techniques have been avoided and the principles of sampling have been simplified, so as to make them practicable to a great extent. Particular emphasis is laid on the interpretation of the data resulting from sampling in the problem of estimation of lot quality and lot acceptance, although many of the concepts and definitions, principles of sample selection, etc are equally applicable to problems connected with process control, industrial design of experiments and so on.

The composition of the Committee responsible for the formulation of this standard is given in Annex B.

Indian Standard

LOT SAMPLING — MANUAL ON BASIC PRINCIPLES

(Third Revision)

1 SCOPE

1.1 This standard explains the various statistical concepts underlying sampling inspection and lays down basic principles of lot sampling. This standard describes sampling inspection techniques applicable for estimation of quality of lot consisting of discrete items or bulk material, and for lot acceptance.

1.2 Specific procedures necessary for sampling a particular material giving details of sampling implements, methods of preparing specimens, etc are not covered in this standard.

1.3 Detailed guidelines and tables for deciding sample sizes and criteria for conformity for lot acceptance are not covered in this standard, and are given various Indian Standards on sampling plans for attributes and for variables.

2 REFERENCES

The standards listed in Annex A contain provisions, which through references in this text constitute provisions of this standard. At the time of publication, the editions indicated were valid. All standards are subject to revision and parties to agreements based on this standard are encouraged to investigate the possibility of applying the most recent edition of these standards.

3 TERMINOLOGY

For the purpose of this standard, the definitions given in IS 7920 (Part 2)/ISO 3534-2 and the following shall apply.

3.1 Acceptance Quality Limit (AQL) — Quality level that is the worst tolerable process average when a continuing series of lots is submitted for acceptance sampling.

3.2 Coefficient of Variation (v) — The absolute value of the ratio of the standard deviation to the average. The coefficient of variation may also be expressed as a percentage by multiplying this ratio by 100.

3.3 Confidence Coefficient — The performance characteristic 100 $(1 - \alpha)$ percent, where, α is generally a small number, that the confidence interval would contain the true parameter value. It is also called confidence level. The confidence level is often 95 percent or 99 percent.

3.4 Confidence Interval — Interval, bounded by a lower limit statistic and an upper limit statistic (T_0, T_1) for the parameter θ for which it holds that $P [T_0 < \theta < T_1] \geq 1 - \alpha$.

NOTES

1 The confidence reflects the proportion of cases that the confidence interval would contain the true parameter value in a long series of repeated random samples under identical conditions. A confidence interval does not reflect the probability that the observed interval contains the true value of the parameter (it either does or does not contain it).

2 Associated with this confidence interval is the attendant performance characteristic 100 $(1 - \alpha)$ percent, where, α is generally a small number. The performance characteristic, which is called the confidence coefficient or confidence level, is often 95 percent or 99 percent. The inequality $P [T0 < \theta < T1] \geq 1 - \alpha$ holds for any specific but unknown population value of θ .

3.5 Consumer's Risk Quality (CRQ) — Lot or process quality level that in the sampling plan corresponds to a specified consumer's risk.

3.6 Gross Sample (Bulk Material) — Aggregation of all the increments taken from a sub-lot or lot by the procedures of routine sampling.

3.7 Hundred Percent Inspection — Inspection of selected characteristic(s) of every item in the group under consideration.

3.8 Inspection — Activity such as measuring, examining, testing or gauging one or more characteristics of a product or service, and comparing the results with specified requirements in order to establish whether conformity is achieved for each characteristic

3.9 Inspection by Attributes — Inspection whereby either the item is classified simply as conforming or nonconforming with respect to a specified requirement or set of specified requirements, or the number of nonconformities in the item is counted

NOTE — Inspection by attributes includes inspection for conformity of items as well as inspection for number of nonconformities per hundred items.

3.10 Inspection by Variables — Inspection by measuring the magnitude(s) of the characteristic(s) of an item.

3.11 Item — Anything that may be described and considered separately, for example, a physical item; a defined amount of bulk material; a service, activity, person, system or some combination thereof.

3.12 Lot (Bulk Material) — Definite part of a population, comprised of the total amount of bulk material under consideration, and where this part is considered as an amount of material for which specific characteristics are to be determined.

NOTE — Commerce in a bulk material often encompasses transactions involving a single lot and, in these cases, the lot becomes the population.

3.13 Lot (Discrete Items) — Definite part of a population constituted under essentially the same conditions as the population with respect to the sampling purpose.

NOTE — The sampling purpose may, for example, be to determine lot acceptability, or to estimate the mean value of a particular characteristic.

3.14 Lot Mean (μ) — Sum of the observations or test results on all items divided by the total number of items in the lot.

3.15 Lot Standard Deviation (σ) — The square root of the mean of the squares of the deviations of all observations or test results in a lot from their mean.

3.16 Nonconforming Item — Item with one or more nonconformities.

3.17 Nonconformity — Non-fulfilment of a specified requirement.

3.18 Range (*R*) — The difference between the largest and the smallest observations or test results in a sample.

3.19 Sample — Subset of a population (lot) made up of one or more items or sampling units intended to provide information about lot.

3.20 Sample Average ($\bar{x}$) — Sum of the observations or test results divided by the number of items in the sample.

3.21 Sample Standard Deviation (*s*) — The square root of the quotient obtained by dividing the sum of squares of deviations of the observations or test results from their mean by one less than the number of observations in the sample.

3.22 Sampling Inspection — Inspection of selected items in the group under consideration.

3.23 Sampling Plan — Combination of sample size(s) to be used and associated lot acceptability criteria.

3.24 Sampling Unit (Bulk Material) — One of the member parts, each with equal probability of selection in sampling, into which a population, comprised of the total amount of bulk material under consideration, is divided:

NOTES

1 In bulk sampling, the operative characteristics of the sampling unit are that the probability of all sampling units is equal and that the entire sampling unit becomes part of the sample when it is selected.

2 When sampling from a bulk material is performed by extraction of individual increments, the sampling unit is the primary increment.

3.25 Sub-lot (Bulk Material) — Definite part of a lot of bulk material.

4 SYMBOLS

The symbols used in this standard are given below:

AOI	average amount of inspection per lot
AOQ	average outgoing quality
AOQL	average outgoing quality limit
AQL	acceptance quality limit
ASN	average sample number
Ac	acceptance number
CRQ	consumer's risk quality
c	number of nonconformities in an item
$\bar{c}$	average number of nonconformities in a sample
$\bar{c}'$	average number of nonconformities in a lot
c_4	factor for estimating population standard deviation (σ) from sample standard deviation s. Values of c_4 are given in Table 1
d	number of nonconforming items in a sample
d_2	factor for estimating population standard deviation (σ) from sample range R or $\bar{R}$. Values of d_2 are given in Table 1

f_1 and f_2	factor for *s* for estimating the confidence limits of σ. Values of f_1 and f_2 are given in Table 7
f_3 and f_4	factors for *R* for estimating the confidence limits of σ. Values of f_3 and f_4 are given in Table 8
h	coefficient of R or $\bar{R}$ for estimating the confidence limits of µ. Values of *h* are given in Table 6
h'	coefficient of s for estimating the confidence limits of µ. Values of h' are given in Table 5
k	factor associated with sample standard deviation *s* for deciding acceptability of lot under variables sampling plan
L	lower specification limit
LQL	limiting quality level
m_1, m_2	the distance of acceptance-rejection lines from origin on the y-axis in the case of sequential sampling plans
N	lot size
n	sample size
OC curve	operating characteristic curve
p	fraction nonconforming
p´	lot fraction nonconforming
R	range
$\bar{R}$	average range
Re	rejection number
s	sample standard deviation
U	upper specification limit
µ	Population mean
v	Coefficient of variation.
σ	population (lot) standard deviation
$\bar{x}$	sample average

5 CLASSIFICATION AND SPECIFICATION OF QUALITY CHARACTERISTIC

5.1 Classification of Quality Characteristic

5.1.1 Quality characteristics are broadly classified into the following three types, according to the method of inspection:

a) *Attributes* — Each item inspected is classified into either of the two categories depending upon presence or absence of the characteristic(s):

Examples:

Sl No.	*Attributes*	*Category I*	*Category II*
(1)	(2)	(3)	(4)
i)	Colour of sugar	White	Not white
ii)	Existence of blow-holes	With holes	Without holes
iii)	Conformity of bolt diameter to the specification	Conforming	Nonconforming
iv)	Grain size	Passing through the sieve of specified size	Not passing through the sieve of specified size

b) *Number of nonconformities* — Number of nonconformities on each inspected item are counted, such as the number of blisters/bubbles on test tubes, number of end-breaks per unit length of yarn; and

c) *Variables* — Each item inspected gives rise to a measurement of a characteristic on a continuous scale, for example, tensile strength of steel wires, specific gravity of paint, etc.

5.1.2 Variables type and number of nonconformities type of data may be converted to attributes type of data with the help of any specification on the measurable characteristic or limit on the number of nonconformities. Thus, a bolt of diameter 11.5 mm would be classified as 'good' if the specification stipulates a minimum diameter of 11.0 mm, or a copper sheet having no nonconformity would be classified as 'good' if the specification stipulates a maximum of one nonconformity. However, it may be noted that any such conversion of the variables or number of nonconformities type of data to the attributes type results in a certain loss of information.

5.2 Specification and Measures of Quality

5.2.1 Whether it is a lot or a sample which is subjected to inspection, ultimately the inspection is performed on the item for a characteristic. When quality characteristic is of the attributes type, it is specified by means of a standard specimen or in terms of specification limits or as a result of some tests. For example, a phial of sugar with certain whiteness for inspecting colour of sugar, or a pair of “Go” “No-Go” gauges corresponding to tolerances for the diameter of bolt or the presence of negative or positive reaction for testing the genuineness of honey or passing or failing of heating elements in high voltage test.

5.2.2 When the quality characteristic is of the number of nonconformities type, only an upper limit is specified. For example, the number of dead pin knots per face of plywood panel should not be more than 4.

5.2.3 When quality characteristic is of the variables type, it is specified in terms of limit values, depending on the quality characteristic, there will ordinarily be either only one (lower or upper) or two (lower and upper) specification limits. For example, it may be specified that the moisture and volatile matter in shaving soaps shall not be more than 12 percent by mass, the elongation percent of aluminum conductors shall not be less than 25; nicotine content in cigarettes shall be between less than 3.5 percent by mass.

5.2.4 *Lot Quality*

Whichever be the type of quality characteristic, the corresponding lot quality may be described in terms of the percentage (or fraction) of nonconforming items. When the quality characteristic is of the attributes type, the only measure of lot quality is in terms of fraction (or percent) nonconforming. When the quality characteristic is of the number of nonconformities type, the lot quality may also be expressed in terms of number of nonconformities per item or for 100 items.

When the quality characteristic is a variable type, the lot quality may also be expressed in terms of the average and standard deviation (or range) values of the variable characteristic based on test results of all items in the lot (standard deviation is a common measure of variability).

5.2.5 *Sample Quality*

So far as a sample, like a lot, contains more than one item, sample quality will be judged from the quality manifestations of the different items constituting the sample. Corresponding to each of the quality characteristic, there will be a sample quality measure. Thus, for attributes type of quality characteristic, a sample percentage nonconforming is invariably used. For number of nonconformities type of quality characteristic, sample quality would be the average number of nonconformities of all the items in the sample. For variables type of quality characteristic, sample quality would be the average and variability (standard deviation or range) of test results of quality characteristic of all the items in the sample.

6 SAMPLING INSPECTION

6.1 Comparison of Hundred Percent Inspection and Sampling Inspection

A brief comparison of the relative advantages and disadvantages of hundred percent inspection and sampling inspection are enumerated as follows:

Sl No. (1)	*Hundred Percent Inspection* (2)	*Sampling Inspection* (3)
i)	Total cost of inspection per lot is high, sometimes prohibitive, if the test or analysis involved is costly	Total cost of inspection per lot is low
ii)	Quite time consuming, if the lot size is large	Decisions may be arrived at quickly
iii)	Generally, not feasible due to limitations of skilled personnel, complex instrumentation, etc	Generally feasible
iv)	More handling damage during inspection	Less handling damage during inspection
v)	Complete accuracy of inference is seldom attained due to inspection errors arising out of fatigue, negligence, difficulty of supervision, etc	Inspection may be organized and controlled more efficiently. Reasonable accuracy of inference is possible. Also measure of sampling error may be obtained

Table (*Concluded*)

Sl No. (1)	*Hundred Percent Inspection* (2)	*Sampling Inspection* (3)
vi)	Not feasible when the inspection involves destructive tests (for example, testing life of bulb, breaking strength)	Only method possible when the inspection involves destructive tests
vii)	The supplier has little incentive to improve his products, since only nonconforming items are rejected and sent back to him for replacement	As the lot itself may be rejected based on sampling results and all items including good ones will be returned, so, supplier tries to improve quality
viii)	The method available when no risk or error can be allowed. But in such cases also, more than 100 percent inspection may be required, like each stage being checked by two independent persons	Some risk (measurable), due to sampling error, will have to be tolerated. So not feasible in cases where no risk is allowed

It may be seen from above that sampling inspection has many advantages and has wider practical applications in industry as compared to hundred percent inspection. However, when hundred percent inspection becomes inevitable, it is advisable to superimpose a sampling inspection, so as to check the efficacy of hundred percent inspection.

6.2 Inspection by Attributes or Variables

6.2.1 When the items are inspected for their quality and results are expressed as attributes, the procedure is called ‘sampling inspection by attributes’. For further details, [*see* IS 2500 (Part 1)/ISO 2859-1].

6.2.2 When the items are inspected for their quality and results are expressed as number of nonconformities, the procedure is called ‘sampling inspection by number of nonconformities’. For further details, [*see* IS 2500 (Part 1)/ISO 2859-1].

6.2.3 When items are inspected for their quality and results are expressed in terms of units of measurement on a continuous scale, the procedure is called ‘sampling inspection by variables’. For further details, (*see* IS/ISO 3951-1).

6.2.4 *Comparison of Inspection by Attributes and by Variables*

When a choice between attributes inspection and variables inspection is relevant, as when a characteristic may either be measured or gauged, the following are some of the important considerations providing a basis for the appropriate choice:

Sl No. (1)	*Attributes Inspection* (2)	*Variables Inspection* (3)
i)	Since the inspection is performed either visually or by gauging, the cost of inspection per item is low.	Since the inspection is done by measurement and the test results may have to be recorded, it involves more time and labour. The cost of inspection per item is, therefore, high.
ii)	For the same degree of efficiency in inference to be made about the lot quality, more items need be inspected.	For the same degree of efficiency in inference to be made about the lot quality, less items need be inspected.
iii)	Computation of sample quality is less complicated.	Computation of sample quality is more complicated.
iv)	Inspection may be subjective, especially in respect of visual characteristics.	Inspection is more objective and minimizes the possibilities of misclassification.
v)	Broad based and has wider applications.	Based on assumption that the characteristic under consideration n has a normal distribution.

Table (*Concluded*)

Sl No.	*Attributes Inspection*	*Variables Inspection*
(1)	(2)	(3)
vi)	More than one characteristic may be considered at a time, finally resulting in classification of the item as nonconforming or conforming.	Only one characteristic may be taken into account at a time.

7 PRINCIPLES OF SAMPLE SELECTION

7.1 Sampling Error

The logic of sampling is that of generalizing from a portion about the whole; on the basis of evidence gathered from a sample drawn from a lot, an inference is made about the lot. Sampling then is a short-cut method for investigation of lots and naturally offers considerable economy in time and labour. Against this saving, however, is the error characteristic of all generalizations. The nature of this error would be different for different problems. Since a sample constitute only a part of the lot, many samples may be drawn from a lot. The inference about the lot quality may be different for different samples. Such differences constitute sampling errors.

7.2 Factors Affecting Sampling Error

The sampling error will depend on the degree of homogeneity of the lot as well as the relative size of the sample. The more the homogeneity in the lot or larger the size of the sample, the smaller would be the sampling error. The method of sample selection also contributes to sampling error.

7.3 Methods of Sample Selection

7.3.1 *Bias in the Selection of Items in Sample*

Bias in the selection of items in a sample units may considerably affect the sampling error and subsequently sampling efficiency. Such bias in sampling may generally arise in the following ways:

a) Certain positions in the lot may be given preference, with the result that bias in sampling will be introduced if the position of items in the lot is not independent of the quality characteristic. For example, good articles may be more at the top in a container; machined parts coming out of lathe, at different intervals of time, may differ in dimensions due to tool-wear; and larger particles may be at the bottom in a conical heap of sand dropped from a chute;

b) Because of the existence of variability in quality characteristic, a selection procedure may be such as to give preference for certain item qualities. For example, an inspector may be able to spot nonconforming items quickly and may be inclined either to collect more of them or avoid them in the sample; and

c) Certain short cut methods for collection of sample or ignorance on the part of the sampler may also lead to a biased sample. For example, in sampling loaded wagons of iron ore, the samples collected from the top surface only may render the sample unrepresentative, or in picking articles from the container more of the bigger ones which have a higher value of the characteristic under consideration may be selected.

7.3.2 *Random Sampling*

Random sampling is the method of sampling wherein a sample is drawn in such a manner that the chance for inclusion of any item in the sample is predetermined. Such a chance is independent of the quality characteristic of the item and its location in the lot. When the chance for inclusion of any item in the sample is same, it is referred to as simple random sample. The procedure to collect a proper simple random sample is to use some mechanical device or its equivalent which will eliminate bias in selection. For instance, the items in the lot might be serially numbered and a device like the wheel of chance used or each number be represented on different cards which are shuffled well and the requisite number of cards selected blindly. Thus, providing the serial numbers of items which form the sample. Alternatively, a table of random numbers, for example, the one given in IS 4905 may be used. Practicable schemes of any randomizing procedure may be devised to suit the configuration of the lot, other physical circumstances. the quality characteristic, the inspection operation, etc.

7.3.3 *Systematic Sampling*

When the items are presented in an orderly manner, it is quite convenient to select items systematically at regular intervals of position or time. Initially, one item is chosen from the lot at random and, thereafter, items are selected regularly at pre-determined intervals till the requisite number of sample items are collected. In practice, this method has been found to be quite a good approximation to simple random sampling described in 7.3.2.

7.3.4 *Stratified Sampling*

When a lot consists of items which may be divided into a certain number of more homogeneous groups or sub-lots or strata, the method of stratified sampling becomes the obvious choice. According to this method, each sub-lot or stratum is sampled separately, so as to obtain a sample representative of the entire lot. It has been found that the sample obtained by this method of sampling is more efficient than a simple random sample drawn from the entire lot in the sense that sampling error will be less in the former case. The more the homogeneity of the sub-lots, the more efficient would be the stratified sampling. When the sub-lots are of different size, the proportional allocation of sample size to different sub-lots becomes is normally done. Even though there is no prior information for dividing a lot into sub-lots on the basis of the homogeneity of the sub-lots, this sub-division is adopted for assisting in the collection of a representative sample from the lot. It is precisely for this reason that sub-division principle is being widely followed in the sampling of all ores and raw materials.

7.3.5 *Sampling in Stages*

There are situations in which the lot is presented in a large number of sub-lots or packages from which neither a random sample of items from the entire lot nor a proportional sample of items from each sub-lot or package may be feasible. For example, in the case of a lot containing packages it would often mean the opening of all the packages or, alternatively, mixing the items in the packages together before selecting the items. The cost of locating, opening and sampling of packages is generally many times that of inspecting an individual item, making it necessary that the maximum number of items from the minimum number of packages should be inspected, without, of course, losing sight of the sampling efficiency. In such cases, instead of sampling from all the packages. a random sample of packages is first chosen and from each package selected a random sample of items proportional to size of the package is chosen. This procedure may be extended to more than one stage. For example, high permeability nickel iron is delivered in rolls weighing several thousand kilograms. A primary sample-unit will be a heat, a secondary sample-unit will be a roll and a tertiary sample-unit will be a specimen (item) from within a roll. Again in the case of coal, a primary sample- unit may be the mine; a secondary sample-unit, a truck-load; and a tertiary sample-unit, an increment from the truck-load.

7.3.6 For further details of above methods and some other methods of random selection, relative merits and demerits of all these methods reference may be made to IS 4905. However, in this standard, for purposes of making inference dealt with in 9 and 10, no distinction will be made between samples on account of differences in method of selection of sample-units; samples of the same size, whatever be the way in which the units in it have been selected, will be considered similar and equivalent to the simple random sample that may be obtained from the lot.

8 PLANNING SAMPLE SELECTION

8.1 Preliminaries to Sample Selection

8.1.1 In order that the assumptions in sampling theory are not invalidated and that the maximum possible sampling efficiency is achieved, proper planning of sample selection is necessary. For planning purposes, as much information about the lot (such as the nature of items and the quality characteristics) as available prior to inspection of the items will be useful. The important preliminaries to sample selection are:

a) Purpose of sample selection, that is, estimation of lot quality or lot acceptance;

b) Define what forms the lot and its size (*see* 8.2);

c) In case of bulk material, choice of the sample unit including its size (*see* 8.3);

d) Sample size to be taken from the lot (*see* 8.4);

e) Method of sample selection (*see* 7.3):

f) Choice of quality characteristic (specifying a non-nonconforming item, if relevant);

g) Choice of lot quality (specifying acceptance quality limit); and

h) Criteria to be used for making the inference (*see* 9 and 10).

8.1.2 The next step would be to provide a sampling plan to suit the purpose in view. In order to provide a sampling plan, some stipulations as regards the permissible limits of errors in inference due to sampling will be necessary. The nature of stipulations will be different for different types of inference, whether assessing lot quality or lot acceptance.

8.2 Formation of Lots

8.2.1 Formation of lots will have to be planned with respect to the following:

a) Type of lot (stationary lot or moving lot, item-lot or bulk-lot);

b) Size of the lot;

c) Homogeneity of the lot, and of sub-lots; and

d) Identity of lots and accessibility to items in the lot.

8.2.2 *Stationary and Moving Lots*

From a stationary lot, in which the items are presented simultaneously, all the sample items may be obtained at one time. In a moving lot, as the lot flows past the point of inspection one or a few items at a time are selected. In many situations stationary lots offer advantages over moving lots, since their identification and disposition are easy and they admit re-sampling and sequential sampling. However, they create stacking-space problem, especially when the items within are to be made easily accessible. But in sampling bulk materials, like coal, grains, molten metal, etc, it is often more convenient to sample while the material is in motion.

8.2.3 *Size of Lot*

The general rule in the formation of lots is to make the lot size as large as possible provided that a reasonable degree of homogeneity is maintained. This is recommended because, for any given degree of efficiency of sampling and for the same degree of homogeneity in the lot, the size required for the sample will not increase as rapidly as the lot size and will not increase after a certain size for a lot (*see also* **10.3** to **10.5**). Thus, a sample of 300 items from a lot of 10 000 items will be more efficient than a sample of 100 items from a lot of 1 000 and will be almost as efficient as a sample of 300 items from a lot of 100 000 items. This means that the cost of inspection per item submitted for sampling inspection will be much less if the lot handled is larger in size. But the size of the lot will have to be limited on account of the following factors:

a) The formation of larger lots may result in the inclusion of items differing more widely in quality;

b) The production or supply of material may be such that the accumulation of large lots will be depleted over a long period;

c) Due to storage and handling difficulties, very large lots will not be feasible; and

d) The economic consequence of rejection of larger lots because of the cost of scrapping, the cost of detailed inspecting or the cost of reworking them.

8.2.4 *Homogeneity of Lots*

As the efficiency of sampling depends on the degree of homogeneity of the lot, efforts should be made not to have a mixed lot as far as possible and to confine the lot to material or products originating from essentially similar conditions, such as raw materials or components of the same source, products manufactured by the same production or assembly line or moulds or patterns, items produced from a single batch or from one setting of the machines during the same day or shift, etc. Product or material coming out of a process under statistical control forms homogeneous lots (*see* relevant parts of IS 397). When, however, sub-lots from different sources are combined to form a lot, as far as possible, the identity of the sub-lots should be preserved, so that efficient stratification may be achieved.

8.2.5 *Identity of Lots and Accessibility to Items in the Lot*

It is also important from the point of view of convenience of inspection that the lot is easily identifiable and is has easy access to all items of the lot to select the representative sample.

8.3 Determination of Sample Size — Bulk Material

8.3.1 When the lot is in bulk material, usually larger the size of increment (area, volume or weight) of the sample, greater would be the sampling efficiency; but beyond a certain size, the rate of gain in sampling efficiency may be negligible. Besides, from practical considerations, the large size of the increment for bulk products like iron ore, coal, etc, may also pose problems like limitations of standard sampling, introduction of errors due to crushing, etc. However, if sampling efficiency is the only criterion for determining the increment size, then that size, beyond which the rate of gain in efficiency would be very small, may be taken as the appropriate item size.

8.3.2 For determining the increment size following this principle, statistically designed experiments should be conducted covering different sizes for the item with respect to the quality characteristic of interest. For each size, from the values of the quality characteristic, a measure of variability is calculated. These values plotted against different sizes for the increment would give a curve. In general, this curve would slope downwards in the direction of increase in the increment size and would be more or less flat after a certain point. The size corresponding to this point would be an appropriate increment.

Example:

> In an investigation conducted on hard coke, large (nominal size 150 mm to 38 mm), 7 series of 50 increments each were collected. The increments in each series were of approximately equal size (weight) which were analyzed by qualified chemists for their ash contents individually. The results of the analyses were obtained over a period of time. The average size of the increment in each series and the corresponding measure of variability (coefficient of variation, *v*) of ash content were as follows:

Sl No.	*Series*	*Average Weight (in kg) of Increment*	*Coefficient Variation Ash Content*
(1)	(2)	(3)	(4)
i)	1	1.7	3.8
ii)	2	2.0	3.5
iii)	3	2.6	3.1
iv)	4	4.0	2.5
v)	5	4.8	2.3
vi)	6	5.2	2.2
vii)	7	6.0	2.1

The coefficients of variation of ash content between the increments have been plotted against the average size of the increments in Fig. 1. On the basis of this investigation. about 5 kg could be taken as a suitable size of the increment. as increments of larger size do not decrease the coefficients of variation appreciably.

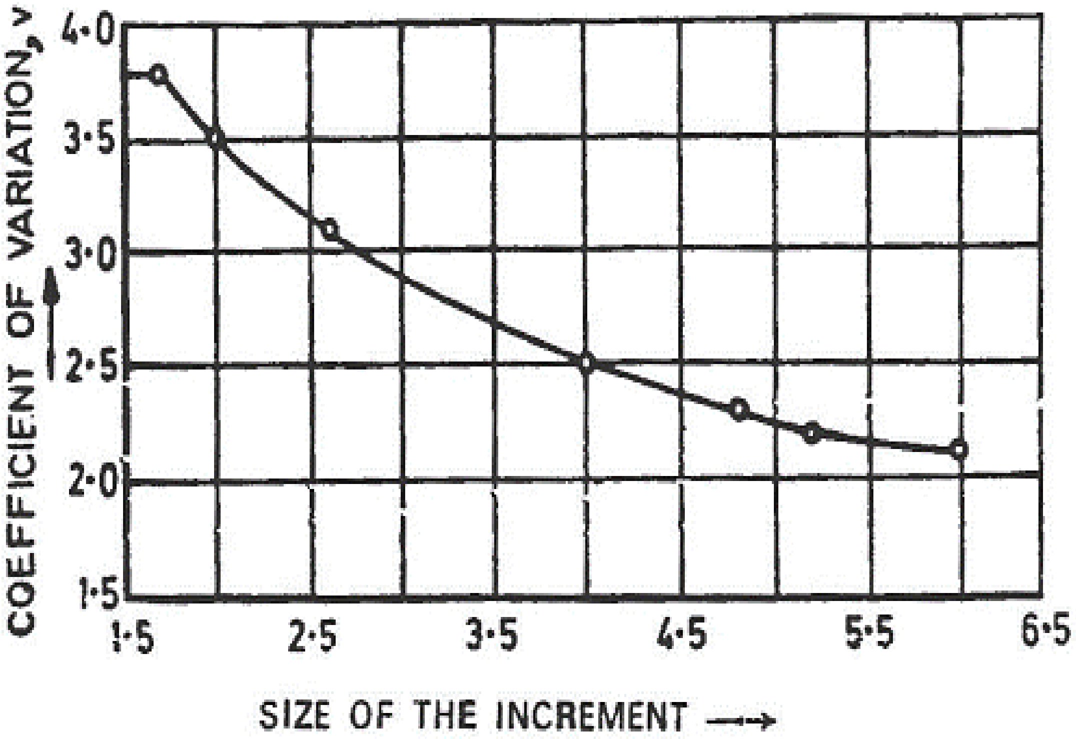


FIG. 1 DETERMINATION OF INCREMENT SIZE

8.4 Determination of Sample Size — Discrete Items

8.4.1 It is not possible to have a single rule for determination of the size of the sample. The number of items to be in the sample is dependent on the extent of error due to sampling that may be tolerated which, in turn, depends to a large extent on the degree of homogeneity of the lot. The size of the lot also comes into picture more from practical considerations than any statistical reasoning, of course, it may be stated that the effect of lot size on sampling efficiency is greater when the lot size is small. But in most industrial usages, where large lot sizes are usually encountered, it is the actual size of the sample that is more important than what percentage sample size forms of the lot. In view of this, specifying the sample as the percentage of the lot size is not recommended (*see* **9.5.1** and **10.1.1**).

8.4.2 For many purposes sample size may have to be large. The conventional assumption that even a small sample could be representative of the lot is to be treated with reserve. The sample size should be determined with due regard to the type of inference, the homogeneity within the lot, size of the lot and with a view to minimizing the cost of inspection. Detailed recommendations for arriving at the desired sample size for estimation of lot quality or for lot acceptance are covered in separate Indian Standards, [*see* IS 5002 and IS 2500 (Part 1)/ISO 2859-1].

9 ESTIMATION OF LOT QUALITY

9.1 General

In estimating lot quality on the basis of a sample, the sample quality consisting of a single value will be adequate for certain purposes. Thus, the sample percentage nonconforming would provide an estimate of the percentage nonconforming in the lot, or the sample arithmetic mean would estimate the arithmetic mean in the lot. But, in view of the error due to sampling, it is often desirable to give an indication of the reliability of the estimate. This is done by providing two limits which are likely to include the true value of lot quality. These two limits, called the upper and the lower limits determine an interval called the confidence interval. This will have a confidence coefficient associated with it. An interval with a confidence coefficient of 95 percent would signify that if, following a given rule, confidence intervals are constructed based on samples of a given size repeatedly drawn from the same lot, then on an average 95 percent of these intervals will contain the true value of lot quality. It is in this sense that it will be ascertained that an interval constructed on the basis of a sample will contain true lot quality. Though confidence intervals for any confidence coefficient like 90 percent, 99 percent, etc, could be computed, the tables given in this standard pertain to a confidence coefficient of 95 percent since it is more widely used in practice.

9.2 Single Value Estimates of Lot Quality

9.2.1 *Lot Fraction Nonconforming*

The sample fraction nonconforming (p) which is obtained by dividing the number of nonconforming items found in the sample by the sample size gives a good estimate of the lot fraction nonconforming (p'). In many situations, this fraction is also expressed as a percentage by multiplying it by 100.

9.2.2 *Average Number of Nonconformities per Item in the Lot*

The average number of nonconformities per item in the sample (c), which is obtained by dividing the total number of all the nonconformities found in all items in the sample by the sample size, gives a good estimate of the average number of nonconformities per item in the lot (c'). In many situations, this average is also expressed as number of nonconformities per 100 items by multiplying it by 100.

9.2.3 *Lot Average* (μ)

When the characteristic under consideration is of the variables type, the sample average ($\bar{x}$), which is obtained by dividing the sum of all the test results by the sample size, is a good estimate of the lot average.

9.3 Single Value Estimates of Lot Standard Deviation (σ)

9.3.1 When the characteristic is of the variables type, calculate sample standard deviation (s). When the sample size is small, say less than 21, the value of the sample standard deviation (s) is to be divided by a suitable factor c_4 given in Table 1 for obtaining an estimate of the population standard deviation (σ). However, for samples of sizes 21 and above, the sample standard deviation itself would give an estimate of the population standard deviation.

Table 1 Factors Necessary for Estimating σ form *s*

(*Clauses* 4 *and* 9.3.1)

Sl No.	Sample Size	Factor c_4
(1)	(2)	(3)
i)	2	0.797 8
ii)	3	0.886 4
iii)	4	0.921 6
iv)	5	0.939 9
v)	6	0.951 1
vi)	7	0.959 3
vii)	8	0.965 0
viii)	9	0.969 6
ix)	10	0.972 5
x)	11	0.975 6
xi)	12	0.977 0
xii)	13	0.979 6
xiii)	14	0.981 2
xiv)	15	0.982 2
xv)	16	0.983 7
xvi)	17	0.984 7
xvii)	18	0.985 4
xviii)	19	0.985 8
xix)	20	0.986 9

9.3.2 If the computation of sample standard deviation (s) from the sample is considered difficult for any reason, the range (R) which is not as efficient as the standard deviation, but simpler to calculate, may be used to estimate σ. The range, however, is a satisfactory substitute for the sample standard deviation only for smaller sample sizes and its use is recommended only if the sample size is less than 10. When range is to be used for large samples, the sample is subdivided into number of subgroups of equal size (less than 10). The average of ranges of all subgroups ($\bar{R}$) is then used instead of R. In most of the quality control practices, subgroup size of 5 is normally used for the sake of convenience. Since the range in the lot by itself is not a common or useful measure of lot dispersion, the sample range R or mean range $\bar{R}$ is used to estimate the lot standard deviation. R (or $\bar{R}$) is divided by a factor d_2 given in Table 2 to get an estimate for σ. The value of d_2 depends on sample size (or subgroup size).

Table 2 Factors Necessary for Estimating σ from *R* or $\bar{R}$

(*Clause* 9.3.2)

Sl No.	Sample Size	Factor d_2
(1)	(2)	(3)
i)	2	1.128
ii)	3	1.693
iii)	4	2.059
iv)	5	2.326
v)	6	2.534
vi)	7	2.704
vii)	8	2.847
viii)	9	2.970
ix)	10	3.078
x)	11	3.173
xi)	12	3.258
xii)	13	3.336
xiii)	14	3.407
xiv)	15	3.472
xv)	16	3.532
xvi)	17	3.588
xvii)	18	3.640
xviii)	19	3.689
xix)	20	3.735

9.4 Interval Estimates of Lot Quality

9.4.1 *Confidence Interval for Lot Proportion Nonconforming* (p')

9.4.1.1 In a sample of size n, if d nonconforming items are found, then 95 percent confidence limits for the proportion of nonconforming items in the lot are given by the two entries at the intersection of the row corresponding to n and the column corresponding to d in Table 3. For example, if in a sample of 20 spark plugs selected from a lot and inspected for high-tension current leakage, 2 were found not passing the test, then the estimated proportion of nonconforming spark plugs in the lot (p') is given by 2/20 = 0.10. The 95 percent confidence limits from Table 3 are 0.018 and 0.293 (that is, 1.8 percent to 29.3 percent).

Table 3 Confidence Limits for Proportion of Nonconforming Items in a Lot with Confidence Coefficient 95 Percent

(*Clause* 9.4.1)

Sl No.	**n**	***d* = 0**	***d* = 1**	***d* = 2**	***d* = 3**	***d* = 4**	***d* = 5**	***d* = 6**	***d* = 7**	***d* = 8**	***d* = 9**	***d* = 10**	***d* = 11**	***d* = 12**	***d* = 13**	***d* = 14**	***d* = 15**
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)
i)	1	.950 .000															
ii)	2	.776 .000	.975 .025														
iii)	3	.632 .000	.865 .017														
iv)	4	.527 .000	.751 .013	.902 .098													
v)	5	.500 .000	.657 .010	.811 .076													
vi)	6	.402 .000	.598 .009	.729 .063	.847 .153												
vii)	7	.377 .000	.554 .007	.659 .053	.775 .129												
viii)	8	.315 .000	.500 .006	.685 .046	.711 .111	.807 .193											
ix)	9	.289 .000	.443 .006	.558 .041	.711 .098	.749 .169											
x)	10	.267 .000	.397 .005	.603 .037	.619 .087	.733 .150	.778 .222										

Table 3 (*Continued*)

Sl No.	**n**	***d* = 0**	***d* = 1**	***d* = 2**	***d* = 3**	***d* = 4**	***d* = 5**	***d* = 6**	***d* = 7**	***d* = 8**	***d* = 9**	***d* = 10**	***d* = 11**	***d* = 12**	***d* = 13**	***d* = 14**	***d* = 15**
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)
xi)	11	.250 .000	.369 .005	.500 .033	.631 .079	.667 .135	.750 .200										
xii)	12	.236 .000	.346 .004	.450 .030	.550 .072	.654 .123	.706 .181	.764 .236									
xiii)	13	.225 .000	.327 .004	.434 .028	.520 .066	.587 .113	.673 .166	.740 .224									
xiv)	14	.206 .000	.312 .004	.389 .026	.500 .061	.611 .104	.629 .153	.688 .206	.794 .206								
xv)	15	.191 .000	.302 .003	.369 .024	.448 .057	.552 .097	.631 .142	.668 .191	.706 .191								
xvi)	16	.178 .000	.272 .003	.352 .023	.429 .053	.500 .090	.571 .132	.648 .178	.728 .178	.728 .272							
xvii)	17	.166 .000	.254 .003	.337 .021	.417 .050	.489 .085	.544 .124	.594 .166	.663 .166	.746 .253							
xviii)	18	.157 .000	.242 .003	.325 .020	.381 .047	.444 .080	.556 .116	.619 .156	.625 .157	.675 .236	.758 .242						
xix)	19	.150 .000	.232 .003	.316 .019	.365 .044	.426 .075	.500 .110	.574 .147	.635 .150	.655 .333	.688 .232						
xx)	20	.143 .000	.222 .003	.293 .018	.351 .042	.411 .071	.467 .104	.533 .140	.589 .143	.649 .209	.707 .222	.707 .293					
xxi)	21	.137 .000	.213 .002	.276 .017	.338 .040	.398 .068	.455 .099	.506 .132	.551 .137	.602 .197	.662 .213	.724 .276					
xxii)	22	.132 .000	.205 .002	.264 .016	.326 .038	.389 .065	.424 .094	.500 .126	.576 .132	.582 .187	.617 .205	.674 .260	.736 .264				
xxiii)	23	.127 .000	.198 .002	.255 .016	.317 .037	.360 .062	.409 .090	.457 .120	.453 .127	.591 .178	.640 .198	.640 .247	.683 .255				

Table 3 (*Concluded*)

Sl No.	**n**	***d* = 0**	***d* = 1**	***d* = 2**	***d* = 3**	***d* = 4**	***d* = 5**	***d* = 6**	***d* = 7**	***d* = 8**	***d* = 9**	***d* = 10**	***d* = 11**	***d* = 12**	***d* = 13**	***d* = 14**	***d* = 15**
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)
xxiv)	24	.122 .000	.191 .002	.246 .015	.308 .035	.347 .059	.396 .086	.443 .115	.500 .122	.557 .169	.604 .191	.653 .234	.661 .246	.692 .308			
xxv)	25	.118 .000	.185 .002	.238 .014	.303 .034	.336 .057	.384 .082	.431 .110	.475 .118	.525 .161	.569 .185	.616 .222	.664 .238	.683 .296			
xxvi)	26	.114 .000	.180 .002	.230 .014	.282 .032	.325 .054	.374 .079	.421 .106	.465 .114	.506 .154	.542 .180	.579 .212	.626 .230	.675 .282	.718 .282		
xxvii)	27	.110 .000	.175 .002	.223 .013	.269 .031	.316 .052	.364 .076	.415 .101	.437 .110	.500 .148	.563 .175	.570 .202	.598 .223	.636 .469	.684 .269		
xxviii)	28	.106 .000	.170 .002	.217 .013	.259 .030	.307 .050	.357 .073	.384 .098	.424 .106	.463 .142	.537 .170	.576 .192	.616 .217	.619 .258	.654 .259	.693 .307	
xxix)	29	.103 .000	.166 .002	.211 .012	.251 .029	.299 .049	.339 .070	.374 .094	.413 .103	.451 .136	.500 .166	.549 .184	.587 .211	.626 .247	.661 .251	.661 .211	
xxx)	30	.100 .000	.163 .002	.205 .012	.244 .028	.292 .047	.324 .068	.364 .091	.403 .100	.440 .131	.476 .163	.524 .175	.560 .205	.597 .236	.636 .244	.676 .292	.676 .324
xxxi)	n	*d* = 0	*d* = 1	*d* = 2	*d* = 3	*d* = 4	*d* = 5	*d* = 6	*d* = 7	*d* = 8	*d* = 9	*d* = 10	*d* = 11	*d* = 12	*d* = 13	*d* = 14	*d* = 15

9.4.1.2 The confidence limits given in Table 3 are correct to three places of decimal and are for $n = 1\ (1)\ 30$ and $d = 0\ (1)\ \left[\frac{n}{2}\right]$. If d is greater than $\left(\frac{n}{2}\right)$, $(n - d)$ would be less than $\left(\frac{n}{2}\right)$. and the Table 3 may be read for confidence limits for the complementary proportion $(1 – p´)$ from which the confidence limits for $p´$ are obtained.

9.4.1.3 *Example*

Suppose, $n = 25$ and $d = 14$. Then $(n - d) = 11$ and the 95 percent confidence limits for $(1 – p´)$ are given as (0.238, 0.664), which means that the 95 percent limits for $p´$ would be (1-0·664, 1- 0.238) = (0·336, 0.762).

9.4.1.4 For sample sizes larger than 30, the confidence limits for the lot proportion of nonconforming items may be calculated as: $p – 1.96\ \{p(1\text{-}p)/n\}^{1/2}$ and $p + 1.96\ \{p(1\text{-}p)/n\}^{1/2}$.

9.4.2 *Confidence Interval for Average Number of Nonconformities per Item in Lot* ($\bar{c}'$)

In a sample of size n, if total of c nonconformities are observed, then the confidence limits at 95 percent confidence coefficient for the average number of nonconformities per item in the lot is obtained by dividing the entries against c in Table 4 by the corresponding sample size n.

Table 4 Values for Determining Confidence Limits for Average Number of Nonconformities Per Item in a Lot (Confidence Coefficient 95 Percent)

(*Clause* 9.4.2)

Sl No.	Total Number of Nonconformities on all Items in Sample	*n* Times Lower Confidence Limit per Item in Lot	*n* Times Upper Confidence Limit per Item in Lot	Total Number of Nonconformities on All Items in Sample	*n* Times Lower Confidence Limit per Item in Lot	*n* Times Upper Confidence Limit per Item in Lot
(1)	(2)	(3)	(4)	(5)	(6)	(7)
i)	0	0.000	3.69	18	10.67	28.45
ii)	1	0.025 3	5.57	19	11.44	29.67
iii)	2	0.242	7.22	20	12.22	30.89
iv)	3	0.619	8.77	21	13.00	32.10
v)	4	1.09	10.24	22	13.79	33.31
vi)	5	1.62	11.67	23	14.58	34.51
vii)	6	2.20	13.06	24	15.38	35.71
viii)	7	2.81	14.42	25	16.18	36.90
ix)	8	3.45	15.76	26	16.98	38.10
x)	9	4.12	17.08	27	17.79	39.28
xi)	10	4.80	18.39	28	18.61	40.47
xii)	11	5.49	19.68	29	19.42	41.65
xiii)	12	6.20	20.96	30	20.24	42.83
xiv)	13	6.92	22.23	35	24.38	48.68
xv)	14	7.65	23.49	40	28.58	54.47
xvi)	15	8.40	24.74	45	32.82	60.21
xvii)	16	9.15	25.98	50	37.11	65.92
xviii)	17	9.90	27.22			

NOTE — Calculation of sample average number is not necessary. The limits for the average number per item in the lot are obtained by dividing the values in col (2) and (3) by the number n of items in the sample.

9.4.3 *Example*

In order to test the anti-corrosive character of a certain paint, 30 steel panels of the same area were coated with the paint drawn from different parts of the bulk supply and subjected to conditions likely to produce corrosion. The number of rust stains on each panel was counted. The total number of stains on 30 panels added up to 14.

Hence, the average stain per sample is obtained as $c = \frac{14}{30} = 0.47$. The 95 percent confidence limits for the average number of stains per panel in the lot is obtained as $\frac{7.65}{30} = 0.25$ and $\frac{23.49}{30} = 0.78$. Hence, the lower and upper confidence limits for average number of stains per panel (0.47) are 0.25 and 0.78 respectively.

In case the number of nonconformities counted (c) in a sample of size n become more than 50, then the confidence limits for the average number of nonconformities per item in the lot may be calcu1ated as $\bar{c} - 1.96\,(\bar{c}/n)^{1/2}$ and $\bar{c} + 1.96\,(\bar{c}/n)^{1/2}$.

9.4.4 *Confidence Interval for Lot Average Using s or R or $\bar{R}$*

If for a sample of size n, its average is $\bar{x}$ and sample standard deviation is s, then the upper and lower confidence limits for the lot average are given by $(\bar{x} + h'\,s)$ and $(\bar{x} - h'\,s)$ where values of factor h' corresponding to the sample size n obtained from Table 5.

For sample size more than 121, $h' = \frac{1.96}{\sqrt{n}}$

9.4.5 *Confidence Interval for Lot Average Using R or $\bar{R}$*

9.4.5.1 The confidence limits for the lot average may also be computed using the sample range (R) or sample mean range ($\bar{R}$) by using values of factor h from Table 6. The upper and lower confidence limits shall be $(\bar{x} + h\,R)$ and $(\bar{x} - h\,R)$ or $(\bar{x} + h\,\bar{R})$ and $(\bar{x} - h\,\bar{R})$ respectively.

9.4.5.2 The use of sample range (R) is recommended when the sample size is less than 10. For sample sizes above 10, mean range ($\bar{R}$) should be used in place of R. This mean range is calculated by initially dividing the sample items into subgroups of equal size (items in order of production), calculating the range of each subgroup, and then the mean range from these range values.

Table 5 Values of h' for Determining Confidence Limits for the Average of a Measurable Characteristic in the Lot, Using Sample Average and Sample Standard Deviation (Confidence Coefficient 95 Percent)

(*Clauses* . , 9.4.4 *and* 9.4.5.3)

Sl No. (1)	Number of Items in Sample (2)	Value of h' (3)
i)	2	8.984
ii)	3	2.484
iii)	4	1.591
iv)	5	1.242
v)	6	1.05
vi)	7	0.925
vii)	8	0.836
viii)	9	0.769
ix)	10	0.715
x)	11	0.672
xi)	12	0.635
xii)	13	0.604
xiii)	14	0.577
xiv)	15	0.554
xv)	16	0.533

Table 5 (*Concluded*)

Sl No. (1)	Number of Items in Sample (2)	Value of ***h'*** (3)
xvi)	17	0.514
xvii)	18	0.497
xviii)	19	0.482
xix)	20	0.468
xx)	21	0.455
xxi)	22	0.443
xxii)	23	0.432
xxiii)	24	0.422
xxiv)	25	0.413
xxv)	26	0.404
xxvi)	27	0.396
xxvii)	28	0.388
xxviii)	29	0.38
xxix)	30	0.373
xxx)	31	0.367
xxxi)	41	0.316
xxxii)	51	0.281
xxxiii)	61	0.256
xxxiv)	81	0.221
xxxv)	101	0.198
xxxvi)	121	0.180

Table 6 Values of *h* for Determining Confidence Limits for the Average of a Measurable Characteristic in the Lot, Using Sample Average and Range (or Mean Range) (Confidence Coefficient 95 Percent)

(*Clauses* 4, 9.4.5.1 *and* 9.4.5.3)

Sl No.	Number of Items in the Sample, ***n***	Number of Subgroups (m) Each of Size ***n***								
		1	2	3	4	5	6	7	8	9
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
i)	2	6.36	1.72	1.08	0·83	0.70	0·61	0·55	0.50	0·46
ii)	3	1.30	0.64	0.47	0·38	0·33	0·30	0·27	0·25	0·24
iii)	4	0.72	0.41	0.31	0·26	0·23	0·21	0·19	0·18	0·17
iv)	5	0.51	0.31	0.24	0.20	0·18	0.16	0.14	0·15	0·13
v)	6	0.40	0.25	0.20	0.17	0.15	0·13	0·12	0.11	0.11
vi)	7	0.33	0·21	0.17	0.14	0.13	0.12	0.11	0·10	0·09
vii)	8	0.29	0·19	0.15	0.13	0.11	0·10	0·09	0·09	0·08
viii)	9	0.25	0.17	0.13	0.11	0.10	0.09	0·08	0·08	0·07

9.4.5.3 *Example*

With a view to estimating the mean breaking strength of 0.4 percent carbon steel produced in a plant, 8 specimens were tested and the values of their breaking strength (expressed in kg/cm^2) were as follows:

5 780	5 750	5 890	5 910
5 690	5 980	5 950	5 762

The average (= 5839 kg/cm^2) gives a single value estimate for μ, the average breaking strength of the steel. For confidence interval, a measure of variability is also required. The value of sample standard deviation (*s*) works out as 106.49, and Range (*R*) is 290:

a) Using standard deviation, refer to Table 5, the value of *h'* for *n* = 8 is 0.836. The confidence limits are:

   1) Lower confidence limit = (5 839 - 0.836 × 106.49) = 5 750 kg/cm^2; and

   2) Upper confidence limit = (5 839 + 0.836 × 106.49) = 5 928 kg/cm^2.

b) Using range, Table 6 gives for *n* = 8 and m = 1, the value of *h* = 0.29. The confidence limits are:

   1) Lower confidence limit = (5 839 - 0.29 × 290) = 5 755 kg/cm^2; and

   2) Upper confidence limit = (5 839 + 0.29 × 290) = 5 923 kg/cm^2.

Using '$\overline{R}$'

a) In the same above example, if 12 specimens were tested with the following test results:

5 780	5 750	5 890	5 910	5 770	5 970
5 690	5 980	5 950	5 762	5 830	5 730

b) The above 12 test results are regrouped into two subgroups of 6 items each, the first 6 results in first row, and remaining 6 in second row as the other subgroup, then:

$$\bar{x} = 5\,825,\ R_1 = 5\,970 - 5\,750 = 220,$$
$$R_2 = 5\,980 - 5\,690 = 290, \text{ and } \overline{R} = 255$$

c) Referring to Table 6 with *n* = 6 and m = 2 the value of *h* = 0.25. Then confidence limits are:

   1) (5 825 - 0·25 × 255) = 5 761 kg/cm^2; and

   2) (5 825 + 0·25 × 255) = 5 889 kg/cm^2.

NOTE — Confidence limits obtained for the same lot quality may differ to some extent when different sample statistics are used. It should be noted that the meaning given to confidence intervals in **9.1** apply to a uniform procedure (using the same sample quality) and on an average basis. In the long run, but not necessarily in a particular case, standard deviation would give shorter confidence intervals than the range.

9.5 Interval Estimate of Lot Dispersion

9.5.1 *Confidence Interval for the Lot Standard Deviation 'σ' Using 's' or 'R'*

Either the sample standard deviation (*s*) or sample range (*R*) may be used for obtaining the confidence limits for the lot standard deviation. When the former is used, the limits are obtained as $f_1 s$ and $f_2 s$ where f_1 and f_2 are the factors corresponding to the sample size *n* taken from Table 7.

When the range R is used, limits are obtained as $f_3 R$ and $f_4 R$ where f_3 and f_4 are the factors corresponding to sample size n taken from Table 8. Though the values in Table 8 are provided for sample size up to 20, the use of R is recommended for smaller sample size, say less than 10.

Table 7 Factors f_1 and f_2 for Determining Confidence Limits for Standard Deviation of a Measurable Characteristic in the Lot, Using Sample Standard Deviation (Confidence Coefficient 95 Percent)

(*Clauses 4 and* 9.5.1)

Sl No.	Number Items in the Sample	Value of f_1	Value of f_2
(1)	(2)	(3)	(4)
i)	2	0.446	31.91
ii)	3	0.521	6.28

Table 7 (*Concluded*)

Sl No.	Number Items in the Sample	Value of f_1	Value of f_2
(1)	(2)	(3)	(4)
iii)	4	0.566	3.73
iv)	5	0.599	2.87
v)	6	0.624	2.45
vi)	7	0.644	2.2
vii)	8	0.661	2.04
viii)	9	0.675	1.92
ix)	10	0.688	1.83
x)	11	0.699	1.75
xi)	12	0.708	1.7
xii)	13	0.717	1.65
xiii)	14	0.725	1.61
xiv)	15	0.732	1.58
xv)	16	0.739	1.55
xvi)	17	0.745	1.52
xvii)	18	0.75	1.5
xviii)	19	0.756	1.48
xix)	20	0.76	1.46
xx)	21 to 25	0.765	1.44
xxi)	26 to 30	0.784	1.38
xxii)	31 to 35	0.799	1.34
xxiii)	36 to 40	0.811	1.3
xxiv)	41 to 45	0.821	1.28
xxv)	46 to 50	0.829	1.26
xxvi)	51 to 60	0.837	1.24
xxvii)	61 to 70	0.849	1.22
xxviii)	71 to 80	0.858	1.2
xxix)	81 to 90	0.866	1.18
xxx)	91 to 100	0.873	1.17
xxxi)	101	0.879	1.16
xxxii)	$n > 101$	$= 1.96/\sqrt{2n}$	$= 1 + 1.96/\sqrt{2n}$

NOTE — The lower and upper limits are given by $f_1 s$ and $f_2 s$, where, s is the sample standard deviation based on a sample of n items.

Table 8 Factors f_3 and f_4 for Determining Confidence Limits for Variability (Standard Deviation) of a Measurable Characteristic in the Lot, Using Range (Confidence Coefficient 95 Percent)

(*Clauses* 4 *and* 9.5.1)

Sl No.	Number of Items in the Sample *n*	Value of f_3	Value of f_4
(1)	(2)	(3)	(4)
i)	2	0.315	22.3
ii)	3	0.272	1.3

Table 8 (*Concluded*)

Sl No.	Number of Items in the Sample *n*	Value of f_3	Value of f_4
(1)	(2)	(3)	(4)
iii)	4	0.251	1.68
iv)	5	0.238	1.18
v)	6	0.229	0.938
vi)	7	0.223	0.799
vii)	8	0.217	0.709
viii)	9	0.213	645
ix)	10	0.209	0.597
x)	11	0.206	0.561
xi)	12	0.203	0.531
xii)	13	0.201	0.506
xiii)	14	0.198	0.486
xiv)	15	0.196	0.468
xv)	16	0.195	0.453
xvi)	17	0.193	0.44
xvii)	18	0.191	0.428
xviii)	19	0.19	0.418
xix)	20	0.189	0.408

Example:

In the example given under 9.4.5.3 for $n = 8$, Table 7 provides the values 0.661 and 2.04 for f_1 and f_2 to be used with *s*, and Table 8 provides the values 0.217 and 0.709 for f_3 and f_4 to be used with *R*. Thus, confidence limits for σ are given by:

a) When *s* is used:

1) Lower confidence limit = 0.661 × 106.49 = 70.4 kg/cm^2; and

2) Upper confidence limit = 2.04 × 106.49 = 217.2 kg/cm^2.

b) When *R* is used:

1) Lower confidence limit = 0.217 × 290 = 62.9 kg/cm^2; and

2) Upper confidence limit and 0.709 × 290 = 205.6 kg/cm^2.

9.6 Effect of Lot Size on Confidence Interval

In all the earlier paragraphs where the construction of confidence limits for different lot parameters have been discussed, the actual size of the lot had not been taken into account. This has been done since the effect of lot size on the width of the confidence interval is negligible only when sample size is small as compared to lot size (*see* Fig 2). Since, in most of the practical situations, the sizes of the lots encountered are considerable as compared to the sample sizes, the tables given are readily applicable in all such cases.

9.7 Other Uses of Confidence Intervals

Besides providing an interval which contains the lot parameter, in the long run, the confidence limits are also useful for many other purposes. For example, while planning experiments, a rough idea of the lot quality together with the tables for construction of confidence limits may be used to determine how large the sample should be in order to obtain estimates of desired reliability. For further details of the actual methods, (*see* IS 5002). The confidence limits are also useful for testing certain hypothesis about population parameters.

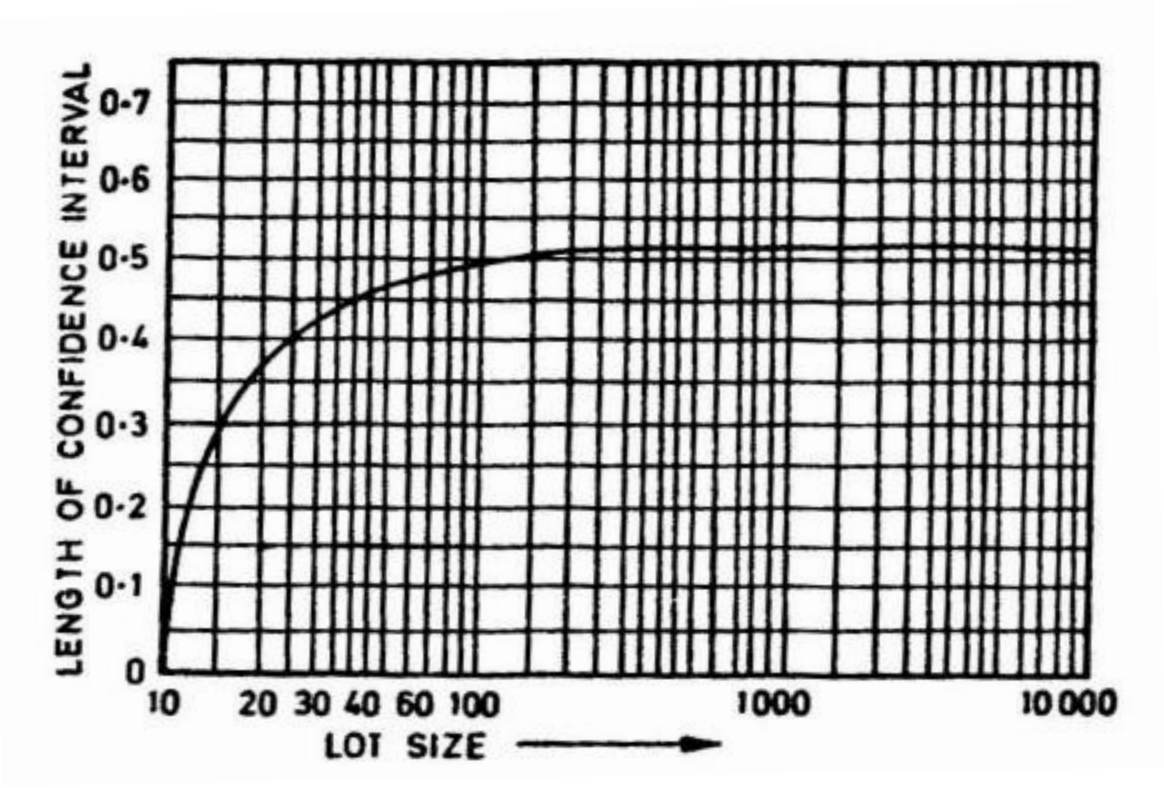


FIG. 2 LENGTH OF CONFIDENCE INTERVAL FOR p CONSTRUCTED FROM n = 10, d = 2 FOR DIFFERENT LOT SIZES

10 SAMPLING FOR LOT ACCEPTANCE

10.1 Risks Involved in Sampling Inspection

In sampling inspection for lot acceptance, a decision is taken about acceptance of the lot or its not acceptance on the basis of outcome of the inspection of sample items. In such a process, wrong decisions by way of either rejecting good lots against the interests of the supplier (or producer) or accepting bad lots against the interests of the receiver (or consumer) are possible. The consumer and producer will, therefore, have to take some risks. It also follows that in lots accepted on a sampling basis, some nonconforming items would be present.

10.2 Operating Characteristic (OC)

10.2.1 When a sampling plan operates and a number of lots are inspected, the result is to sort out the lots into two groups, namely, the accepted and non-accepted groups. The relative proportions into which lots will be sorted out by any particular plan as acceptable or not will, along with other factors, depend upon:

a) sampling plan, that is, sample size (*n*) and acceptance number (that is, permissible number of nonconforming items allowed in sample for acceptance of lot); and

b) the quality of the lots coming for inspection.

10.2.2 The quality of incoming lots might vary. The different proportions into which a sampling plan will, on an average, be able to sort the lots at varying incoming quality levels would, therefore, give a picture of the degree of quality protection given by the sampling plan. For instance, if the consumer fixes a certain level of quality for lots to be considered bad, he may know from the proportion of lots that would be accepted at that quality, whether the sampling satisfies a stipulated risk of accepting bad lots or not. The performance of a sampling plan, measured in terms of proportion of the lots that would be accepted, on an average, at different possible incoming qualities, is called the operating characteristic (OC) of the sampling plan. Each sampling plan will thus have its own OC. The OC of a plan may be expressed as a mathematical function of the incoming lot quality. The graph of this function with incoming lot quality on the horizontal axis is called the QC-curve. If the incoming material is free of any nonconforming items, then the lot will always be accepted, whatever may be the sampling plan chosen. On the other hand, if it contains only nonconforming items, the lot will always be rejected, irrespective of the type of sampling plan chosen. Hence, the OC-curve would continuously slope downwards from the value one to zero in the direction of deterioration of quality, indicating that the plan will have lesser and lesser preference for worse and worse incoming lot qualities. The ability of a plan to discriminate between good and bad lots may be judged from the steepness with which the QC-curve slopes downwards. OC of a plan, effectively summarizes the nature of quality protection obtainable by operating the sampling plan on a number of lots. OC, thus. plays an important role in the choice of a sampling plan and many of the published plans also furnish OC-curves to enable choice from the point of view of quality protection.

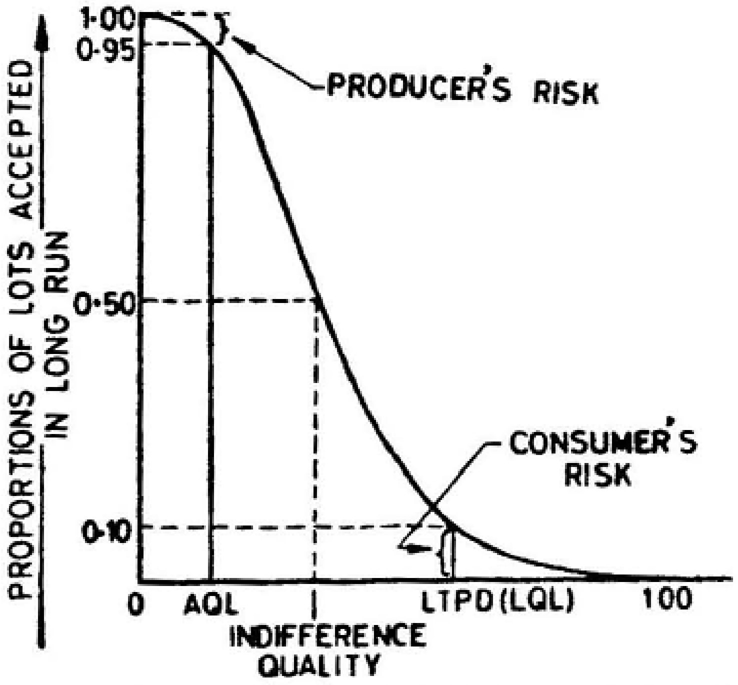
10.2.3 A typical QC-curve is shown in Fig. 3, in which the incoming lot quality is expressed as percentage nonconforming items (the larger the percentage nonconforming, the worse is the quality). The OC-curve may be regarded as a graph of the percentage of lots expected to be accepted in the long run in sampling plan plotted against incoming lot quality.

10.2.4 If a certain value is chosen to demarcate good lots and another to demarcate bad lots, it is easy to identify the producer's and consumer's risk on the OC-curve as shown in Fig. 3. The value demarcating good lots is called the acceptance quality level (AQL) and the value demarcating bad lots is Consumer’s risk quality (CRQ). CRQ is also designated as limiting quality level (LQL). The producer wants all good lots accepted and the consumer wants all bad lots rejected. Only an ideal OC-curve, as shown in Fig. 4. may achieve such an objective. However, sampling plans may have OC-curves close to the ideal OC only, thereby implying that certain risks for both the producer and the consumer are inevitable. In Fig. 3, lots with percentage nonconforming equal to AQL or less will be considered good. Lots with quality equal to CRQ or worse will be considered bad.

10.2.5 Even when the producer submits lots of AQL quality, he runs a small risk of such lots being rejected by the sampling plan. This is known as producer's risk. Similarly, when the lots of CRQ quality are submitted, the consumer runs small risk of accepting such lots, which is known as consumer’s risk. With the shape of the OC-curve, as given in Fig. 3, it would be seen that the risk of rejecting lots of quality better ban AQL will be less than the producer's risk. Similarly, the risk of accepting lots of quality worse than CRQ will be less than the consumer's risk. In practice, producer's risk is normally taken as 5 percent whereas consumer's risk is kept as 10 percent.

10.2.6 The lot quality. which has a 50 percent chance of being accepted or rejected by the plan. is also sometimes used as a measure of protection afforded by the plan. This lot quality is normally known as indifference quality.

NOTE — Though in the development of the theory of the theory of sampling inspection, AQL was associated with a fixed producer’s risk (5 percent), the later trend has been to link it with the attainable process average. Thus, the lots, for which the process average level is not greater than the specified AQL, are accepted most of the times. Accordingly, the producer’s risk is not set at a particular value. For further details, [*see* IS 2500 (Part 1)/ ISO 2859-1 and IS/ISO 3951-1].



INCOMING LOT QUALITY (Percentage Nonconforming)

FIG. 3 AN OC-CURVE

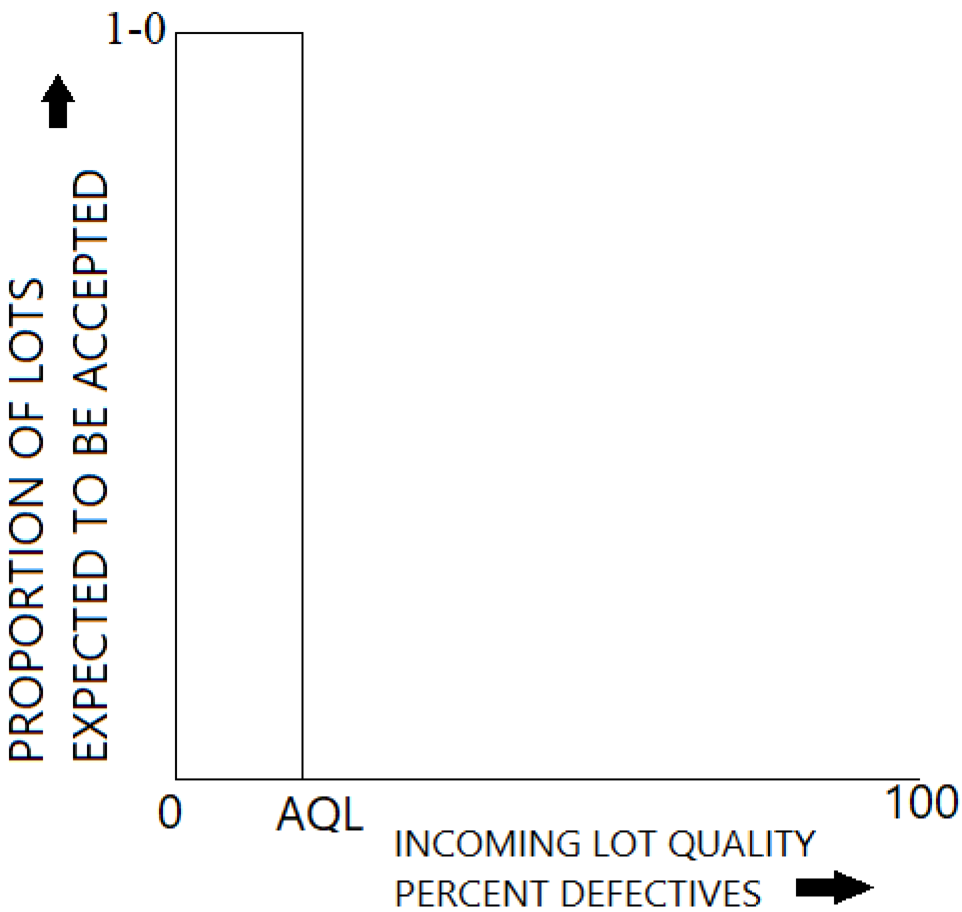


FIG. 4 AN IDEAL OC-CURVE

10.3 Percentage Sampling

A common practice in industry is to specify that the sample shall be some fixed percentage of the lot, such as, 5 percent, 10 percent or 20 percent. This stipulation is generally based on the mistaken idea that the protection given by sampling plans is constant if the ratio of the sample size to lot size is kept constant. It may, however, be shown with the help of OC-curves that acceptance sampling plans with the same percentage samples give very different quality protections. For example, Fig. 5 gives OC-curves for 4 plans wherein the sample is 10 percent of the lot size (lots of size 50, 100, 200 and 1 000 are considered).

It may be seen from Fig. 5 that incoming lots of 2 percent nonconforming items would have a very high chance (about 90 percent) of being accepted when the lots are of size 50 and a very poor chance (about 12 percent) of being accepted when the lots are of size 1 000.

10.4Fixed Sample Size

As against the percentage sampling discussed in **10.3**, it may be of interest to study the behaviour of the plans with fixed sample sizes. Fig. 6 gives the OC-curves of 4 plans wherein a sample of size 20 is inspected (lots of size 50, 100, 200 and 1 000 are considered). It may be easily seen that the fixed sample size tends towards the constant quality protection, since the corresponding OC-curves are close to each other.

10.5 Varying Sample Size

Figure 7 illustrates the change in the shape of OC-curves for different sample sizes keeping the acceptance number constant. As the sample size increases, the slope of the curve becomes steeper and approaches straight vertical line. Sampling plans with large sample sizes are better able to discriminate between good and bad quality lots. Therefore, the consumer has fewer lots of bad quality accepted and the producer fewer lots of good quality rejected.

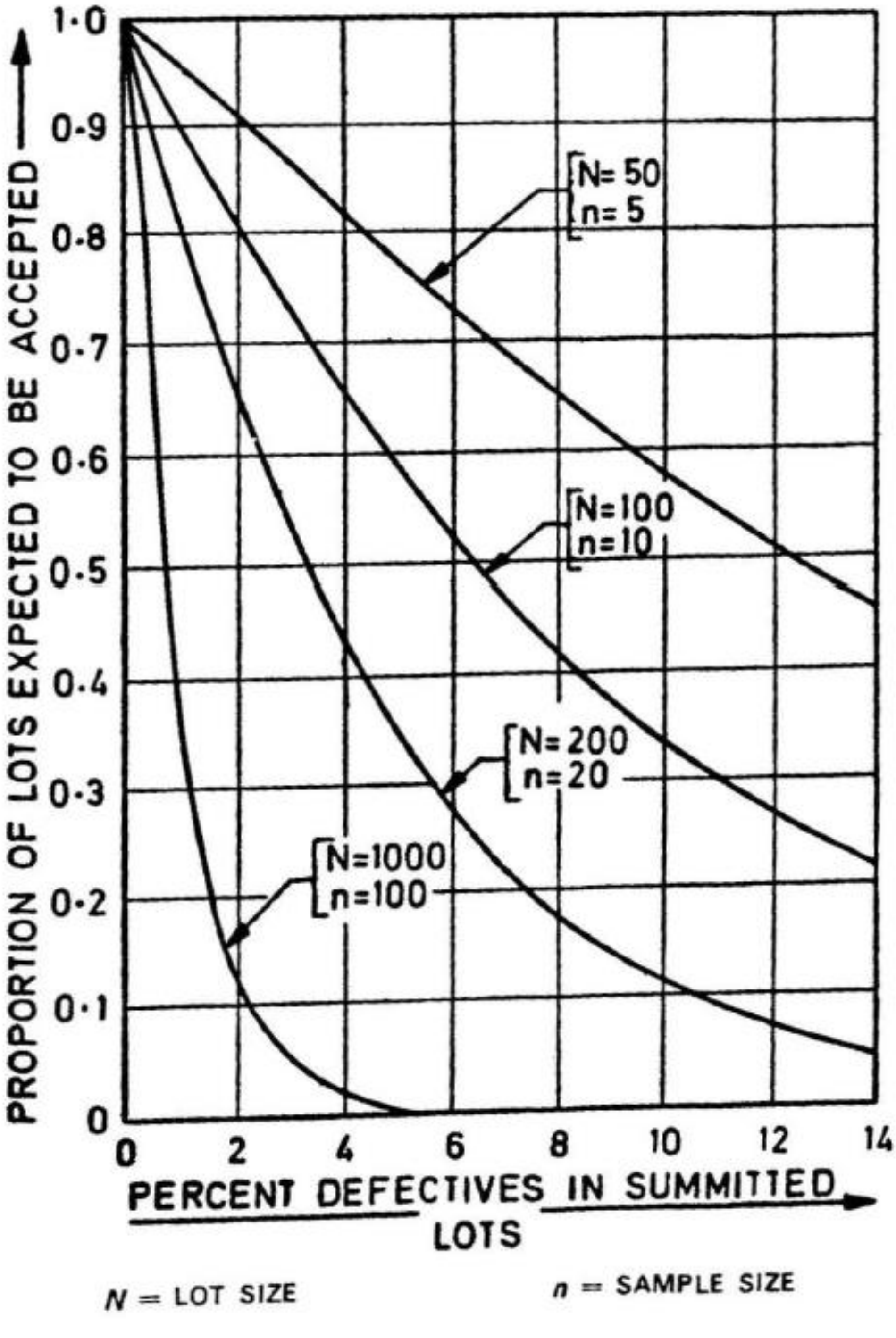


FIG. 5 SOME OC-CURVES FOR PERCENTAGE SAMPLING

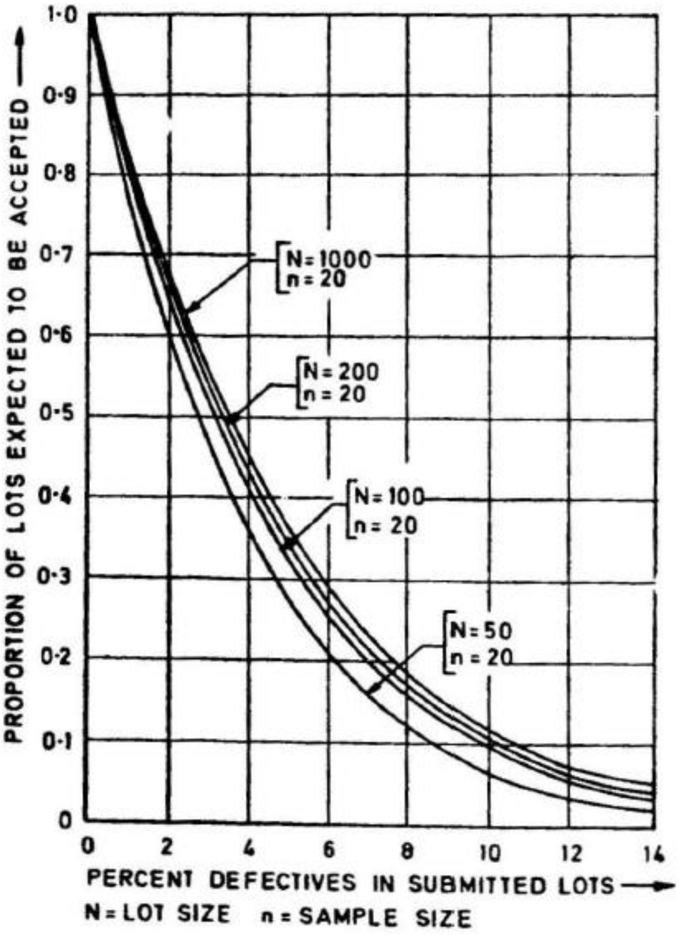


FIG. 6 SOME OC-CURVES FOR PLANS WITH CONSTANT SAMPLE SIZES

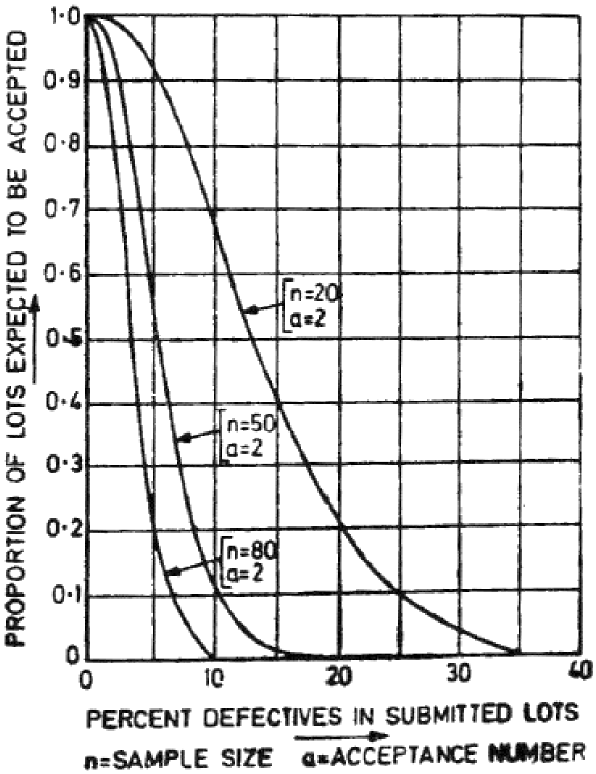


FIG. 7 OC-CURVES FOR VARYING SAMPLE SIZES

10.6 Varying Acceptance Numbers

The changes in the shape of OC-curves as the acceptance number changes keeping the sample size fixed are shown in Fig. 8. As the Acceptance number decreases, the curve becomes steeper. Consequently, the OC-curve for plans with acceptance number zero are the steepest. This fact has frequently been incorrectly used to justify the use of sampling plans with zero acceptance number. It may, however, be seen that OC-curves of such plans are convex throughout the range of incoming quality, resulting in higher risks of rejecting good lots. As against this, OC-curve for *n* = 200 and *a* = 1, which is shown by dotted line in Fig. 8, is not only steeper than the OC-curve for the plan *n* = 50 and a = 0, but also is more concave at the lower values of incoming quality. Thus, the plan for *n* = 200 and a = 1 is more discriminating in the acceptance of AQL quality of incoming material and rejection of CRQ quality as compared to other plans. Sampling plans with acceptance numbers greater than zero may actually be superior to those with zero. Therefore, sampling plans with higher sample sizes and non-zero acceptance numbers may usually be considered better.

10.7 Acceptance Numbers as a Fixed Percentage of Sample Size

Figure 9 illustrates the change in the shape of QC-curves for three different sampling plans having the same ratio of acceptance number to sample size. The three plans are *n* = 20, *a* = 2; *n* = 50, *a* = 5; and *n* = 200, *a* = 20. Although each of these three plans permits 10 percent nonconforming items, it is evident that they have quite different OC-curves. The larger the sample size, the greater the ability of the sampling plan to discriminate the lots of different qualities. For example, a 15 percent nonconforming lot, if submitted for inspection, will have a probability of acceptance as 40·5 percent for the plan *n* = 20, *a* = 2, but is practically certain to be rejected for the plan *n* = 200, *a* = 20. The Large sample, which protects the consumer against the acceptance of relatively bad lots, also gives the producer better protection against the rejection of relatively good ones. For example, a 5 percent nonconforming lot has probability of acceptance as 92·5 percent for plan *n* = 20, *a* = 2 and 99.8 percent for plan *n* = 200, *a* = 20.

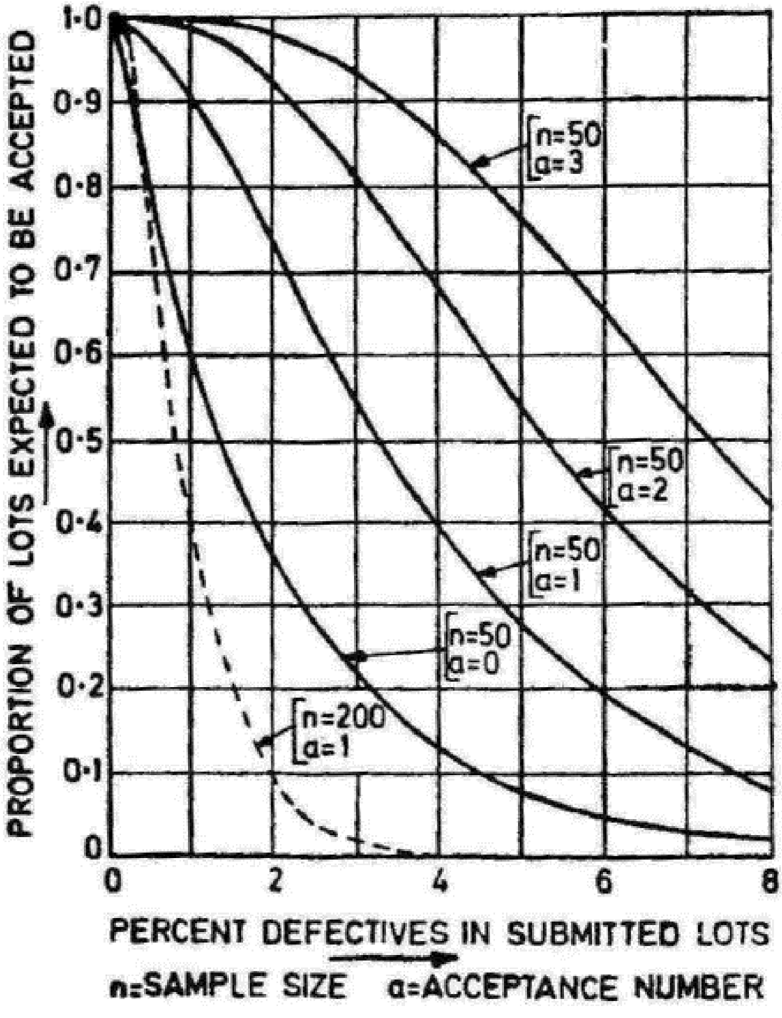


FIG. 8 OC-CURVES FOR VARYING ACCEPTANCE NUMBERS

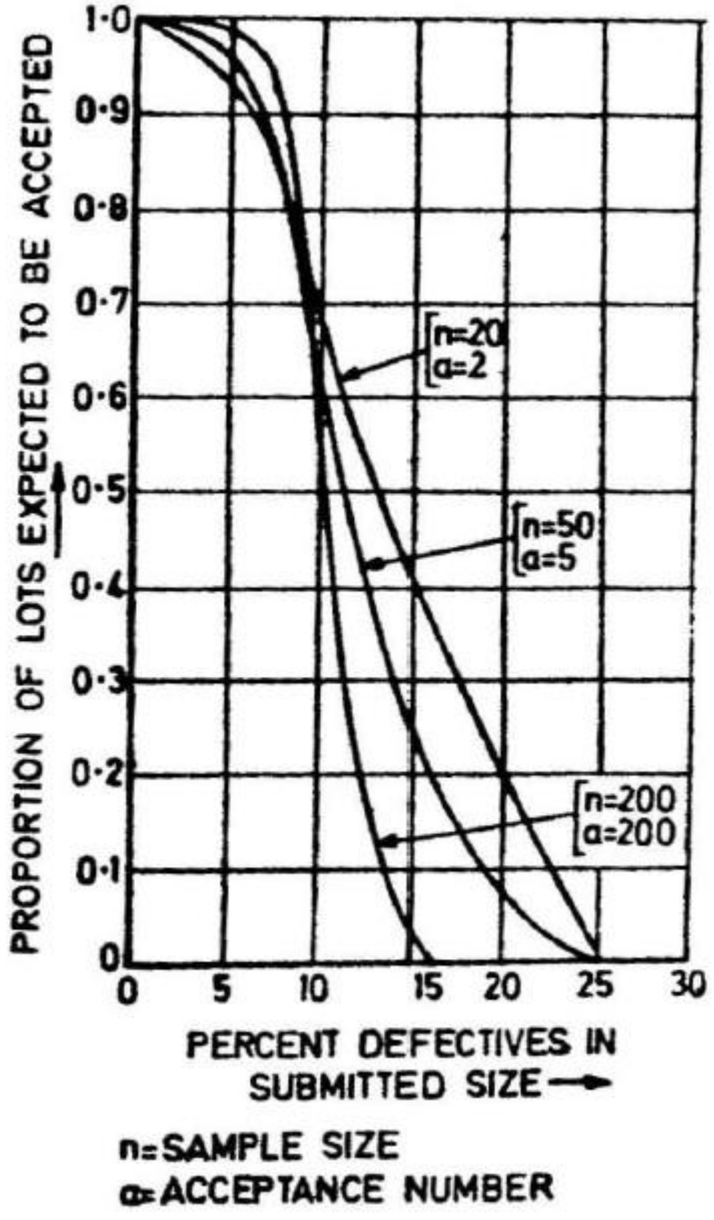


FIG. 9 OC-CURVES FOR CONSTANT VALUES OF $(\frac{a}{n})$

10.8 Average Outgoing Quality Limit (AOQL)

10.8.1 The operating characteristic, while it provides a complete picture of the performance of sampling plan in the form of a function or a curve, does not give a single value measure of the performance. In view of this, other concepts are sometimes used. One such is the average outgoing quality limit (AOQL).

10.8.2 When a plan operates on a number of lots, out of which some will be accepted. The non-accepted lots may be rejected and returned or may be completely inspected for the non-sampled portion of each lot. In the latter case, all the nonconforming items found would be replaced by the good ones. Accordingly, an acceptance plan may be either of the acceptance-rejection type or of the acceptance-rectification type. For both these types, there would be a certain resulting quality for all the accepted lots put together called the average outgoing quality (AOQ). This would naturally depend on the incoming lot quality. If the sampling plan is of the acceptance-rejection type, then the AOQ will be almost the same as the incoming quality (the slight reduction in the value of AOQ as compared to the incoming lot quality is due to the fact that the nonconforming items found in the samples of the accepted lots are eliminated). On the other hand, for an acceptance-rectification type of plan, the AOQ will be superior to the incoming quality since the non-accepted lots will be completely inspected, made free of nonconforming items and then added to the lots accepted at first. Hence, the AOQ concept is more meaningful in relation to the acceptance-rectification type of plans.

10.8.3 If the incoming lots are absolutely free of nonconforming items, the value of AOQ would obviously be zero for any sampling inspection plan. Again, if the incoming lots contain only nonconforming items then all the lots would be rejected by the sampling plan chosen and since rejected lots are to be screened for replacing the nonconforming items by the good items, the value of AOQ would again be zero. For incoming lots of intermediary qualities, the value of AOQ would vary depending on the quality level of the incoming lots. A typical AOQ-curve for the plan in which 50 sample items are inspected from each lot and the lot is accepted if one or less nonconforming is encountered is given in Fig. 10.

10.8.4 The analysis of the curve shows that when the incoming quality is 2 percent nonconforming, the AOQ is 1.48 percent nonconforming and when the incoming quality is 8 percent nonconforming, AOQ is 0.64 percent nonconforming. Since the rejected lots are rectified, the AOQ is always better than the incoming quality. From Fig. 10 it may be easily seen that the AOQ may not exceed a certain limit. This upper limit is called the average outgoing quality limit (AOQL). The AOQL of a plan is, therefore, a summary measure of the quality protection in the case of an acceptance-rectification plan. It should, however, be noted that the AOQL is only an upper limit of an average and it may, sometimes, be exceeded in individual cases. However, in the long run, the percent nonconforming items in the accepted lots would never exceed the AOQL. Thus, an AOQ-curve, in conjunction with an QC-curve, provides a powerful tool for describing and analysing acceptance-rectification type of sampling plans.

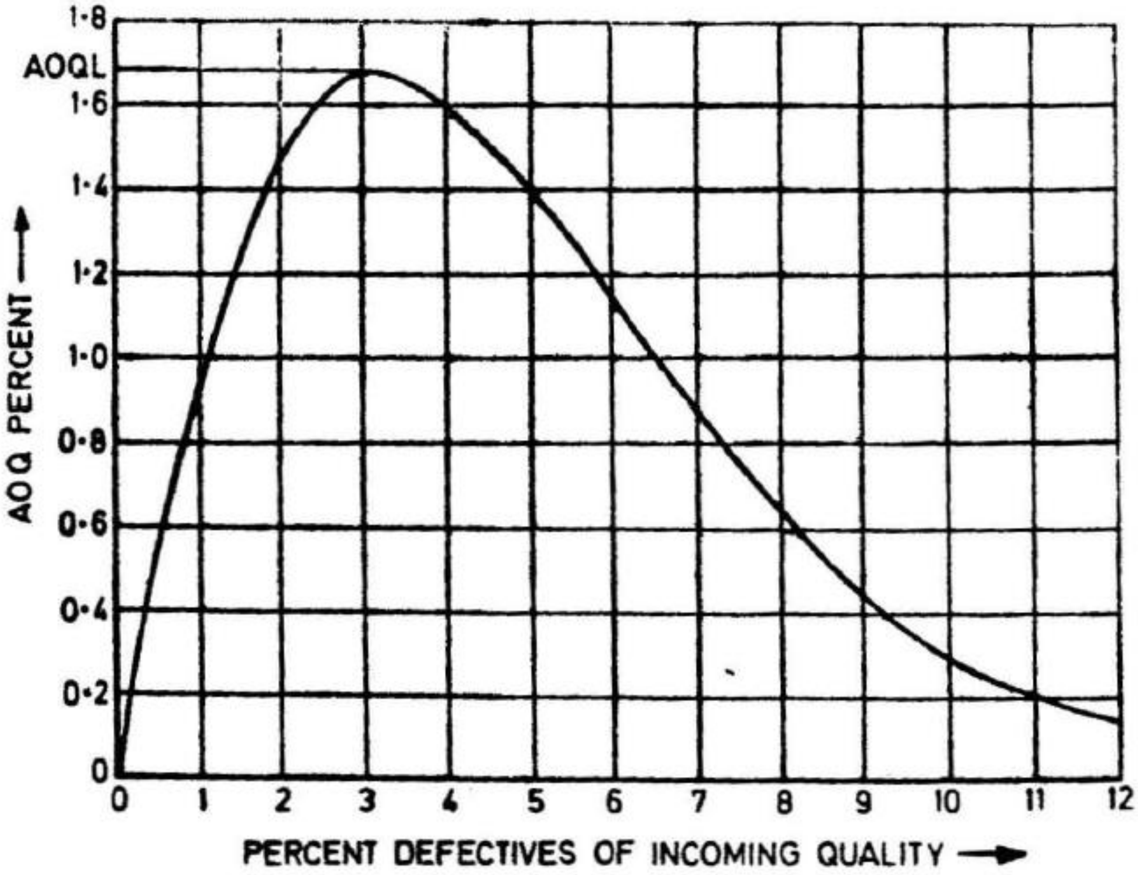


FIG. 10 A TYPICAL AOQ-CURVE

10.9 Different Types of Sampling Plans

10.9.1 Depending upon whether the inspection is by gauging (or visual) or by measurement, the attributes or the variables type of plans may be used. For both these types, the lot quality may be expressed as a percentage nonconforming. Hence, most of the plans (both attributes and the variables type) are designed to suit lot quality specifications in terms of percentage nonconforming items. In the attributes plan. the quality characteristic that is generally used is the proportion of nonconforming items (or nonconformities per item) found in the sample, whereas in the variables plan the quality characteristic is generally expressed is terms of average ($\bar{x}$) and standard (s) or range (R).

10.9.2 *Attributes Plans*

10.9.2.1 *Single sampling plan*

In this type of sampling plans, decision on the acceptance or rejection of the lot is always made on the evidence of only one sample taken from the lot. So, a single sampling plan is defined by the lot size N, the sample size n, and the acceptance number a. If the number of nonconforming items found in the sample is less than or equal to a, the lot is accepted; otherwise the lot is rejected. Hence, rejection number in this case is given by (a + 1) (also denoted by r).

10.9.2.2 *Double sampling plan*

In this type of sampling plans, one sample is taken from the lot and the evidence is used to accept the lot, or to reject it or to reserve decision until further information from a second sample is obtained. Thus, one sample from the lot is always taken and never more than two samples are taken from the lot. So, a double sampling plan is defined by:

Ac = Acceptance number for the first sample

a_2 = Acceptance number for the first and second samples combined

N = Lot size

n = Size of the first or second sample

Re = Rejection number for the first sample

r_2 = Rejection number for the first and second samples combined (= a_2 +1)

Schematically, a double sampling acceptance-rejection plan may be presented as shown in Fig. 11.

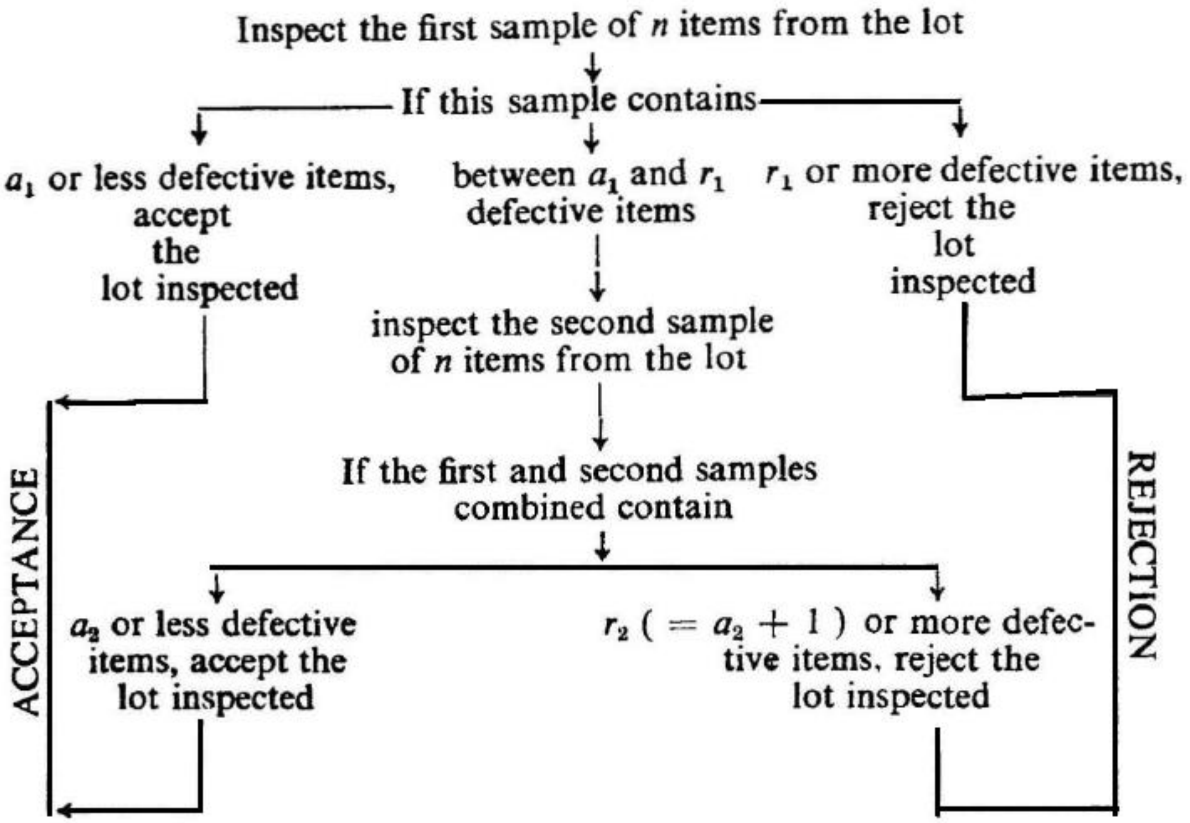


FIG. 11 SCHEMATIC REPRESENTATION OF A DOUBLE SAMPLING PLAN

10.9.2.3 *Multiple sampling plan*

A multiple sampling plan is an extension of the double sampling plan. In this type of plan, a certain number of samples, not exceeding a maximum number of stages specified, are taken before taking a decision on acceptance or rejection of the lot. Although it may be possible to devise sampling plans for any number of stages, the plans published in IS 2500 (Part 1)/ISO 2859-1 are given up to five stages.

10.9.2.4 *Sequential sampling plans*

While in multiple sampling a decision is attempted after each sample is inspected and a decision is arrived at in any case with the last allowable sample, in the sequential type of inspection a decision is attempted on the cumulative evidence gathered at each stage of item-by-item or group-by-group inspection of items. This type of inspection is continued till a categorical decision to accept or reject the lot is possible. There is, therefore, no question of pre-determined sample sizes for sequential plans. Sequential sampling plans lead, on an average, to the least amount of inspection amongst all attributes plans.

10.9.2.5 For detailed information on the following, reference may be made to IS 2500 (Part 1)/ ISO 2859:

a) criteria for selection from amongst various attributes sampling plans;

b) criteria for selection of inspection levels (which determine sample size) and based on these inspection levels, what should be actual sample size from different sizes of lots;

c) criteria for selection of AQLs and hence selection of acceptance and rejection numbers to decide the acceptance or rejection of lot; and

d) criteria for switching from normal inspection to tightened inspection (in case getting poor quality consistently) or reduced inspection (getting good quality consistently).

10.9.2.6 *Examples*

Suppose lots containing 2 000 mild steel tubes each are submitted for inspection of outside diameters. The outside diameters of the tubes should be between 9.8 mm and 10.6 mm. Any tube failing to meet this requirement is considered as nonconforming and suitable gauges are used for the detection of nonconforming items.

If a single sampling plan for with general inspection level II and an AQL of 4 percent is adopted, the sampling plan from IS 2500(Part 1)/ISO 2859 gives sample size (*n*) as 125 and acceptance number (Ac) as 8. Therefore, from each lot of 2 000 tubes, a random sample of 125 tubes should be taken and inspected them for outside diameters using Go, No-Go gauges. The lot shall be accepted if the number of nonconforming tubes found in the sample is 8 or less; otherwise, it shall be rejected.

If a double sampling plan is adopted, the plan would be as follows:

Sl No.	*Sample Stage*	*Sample Size (n)*	.----------*Cumulative* ---------		
			Sample Size	Acceptance No (Ac)	Rejection No (Re)
(1)	(2)	(3)	(4)	(5)	(6)
i)	First	80	80	5	9
ii)	Second	80	160	12	13

From each lot of 2 000 tubes, collect the first sample of 80 tubes at random and inspect them for diameter. The lot shall be accepted if the number of nonconforming tubes in the first sample is 5 or less and rejected if it is 9 or more. If the number of nonconforming tubes is between 5 and 9 (that is, 6, 7 or 8), then a second sample of 80 tubes shall be collected and inspected. The lot shall be accepted if the number of nonconforming tubes in the combined sample of 160 tubes is 12 or less and rejected if it is 13 or more.

If instead of a double sampling plan, a multiple sampling plan is desired to be followed, the sampling plan would be as follows:

Sl No.	*Sample Stage*	*Sample Size*	*For Cumulative Sample*		
			Size	Acceptance Number (Ac)	Rejection Number (Re)
(1)	(2)	(3)	(4)	(5)	(6)
i)	1	32	32	0	5
ii)	2	32	64	3	8
iii)	3	32	96	6	10
iv)	4	32	128	9	12
v)	5	32	160	12	13

From each lot collect the first sample of 32 tubes at random and examine them for outside diameter. The lot shall be accepted if no nonconforming tube is found. It shall be rejected if the number of nonconforming tubes found is 5 or more. If the number of nonconforming tubes is between 0 and 5, a second sample of 32 tubes shall be selected and examined. The number of nonconforming tubes in the combined sample of first and the second sample taken together is then be compared with acceptance number 3 and rejection number 8 corresponding to the second stage of sampling for taking decision. If no decision is reached (number nonconforming in between 3 and 8), a third sample of 32 tubes is drawn and again number of nonconforming tubes compared with acceptance and rejection numbers for the third stage. This process may continue up to maximum fifth stage when a decision is finally reached to accept or reject the lot.

10.9.3 *Variables Sampling Plans*

10.9.3.1 *Single sampling plans*

For detailed information on the following, reference may be made to IS/ISO 3951-1:

a) criteria for selection of inspection levels (which determine sample size) and based on these inspection levels, what should be actual sample size from different sizes of lots; and

b) criteria for selection of AQLs and hence selection of acceptance and rejection numbers to decide the acceptance or rejection of lot.

After determining sample size (n), the sample is selected at random and measurements for the characteristic are obtained on these n items in the sample. Calculate the arithmetic mean ($\bar{x}$) and sample standard deviation (s).

The value of factor k based on AQL of the quality characteristic is selected from IS/ISO 3951-1. In case only upper specification (U) is only specified for the quality characteristic, then the lot is accepted if ($\bar{x} + ks$) is less than or equal to U, otherwise not. When only lower specification limit (L) is specified, then the lot is accepted if ($\bar{x} – ks$) is greater than or equal to L, otherwise not.

When the specification limits for the quality characteristic is two-sided (that is, in terms of both L and U), then for acceptance of lot, it has to be firstly checked if the process is capable to produce items within the specification limits. For this purpose, the value of $\frac{s}{U-L}$ shall also be less than the value of factor f_s specified in IS/ISO 3951-1. If it meets, then acceptability with respect to individual upper and lower specification is checked as mentioned above.

10.9.3.2 *Double sampling and sequential sampling variable plans*

The principle underlying the double sampling and Sequential sampling variables plans are similar to the corresponding attributes plans. Sequential plans are also possible in case of variables inspection which require the determination of acceptance and rejection lines as in the case of attributes sequential plans. However, in practice, these types of plans are very cumbersome to operate and, hence, hardly used in industrial applications.

Example:

> If for the example given in **10.9.2.6**, a single sampling variable plan is to be adopted with variability unknown and estimated by sample standard deviation, then for the same inspection level II and AQL 4 percent, IS/ISO 3951-1 gives sample size of 75 and value of factor k as 1.377. So, a random sample of 75 items is to be taken from the lot and its average ($\bar{x}$) and standard deviation (s) are calculated. As the specification limit is two sided, the value of factor f_s given in IS/ISO 3951-1 is 0.291. The lot shall be accepted for outside diameter if all the following three conditions are satisfied:
>
> a) $s/(U-L) = s/0.8 < 0.291$;
>
> b) $\bar{x} + 1.377\, s < U$; and
>
> c) $\bar{x} - 1.377\, s > L$.

10.9.4 *Average Amount of Inspection*

The cost of inspection involved in using a sampling plan depends on several factors including the number of items required to be inspected. For comparison of costs involved in using different plans, however, often the average amount of inspection per lot (AOI) or the average sample number (ASN) is used. ASN is the average number of items inspected per lot in the sampling inspection whereas AOI is the average number of items inspected per lot in sampling as well as in sorting inspection. ASN and AOI are same in acceptance rejection plans but are much different in acceptance-rectification plans. In a single sampling plan, with sample size n, the ASN is equal to n if the plan is of the acceptance- rejection type. The corresponding ASN for a double sampling plan would be of the form $n\,(1 + f)$ where n is the size of the first or second sample and the fraction of lots for which a second sample is drawn to take a decision for acceptance of the lot. (These average sample numbers hold good if there is no curtailment of inspection, that is, the sample selected is completely inspected, irrespective of the decision to accept or reject the lot).

Typical ASN-curves for single, double and multiple sampling plans having same QC-curves are given in Fig. 12.

In a single sampling plan (acceptance-rectification type) the AOI = n + F (N-n) where, n is the sample size, N the lot size and F the fraction of lots that will not be accepted in the first instance but will be accepted after complete inspection and rectification. For further details, reference may be made to IS 2500 (Part 1)/ISO 2859-1.

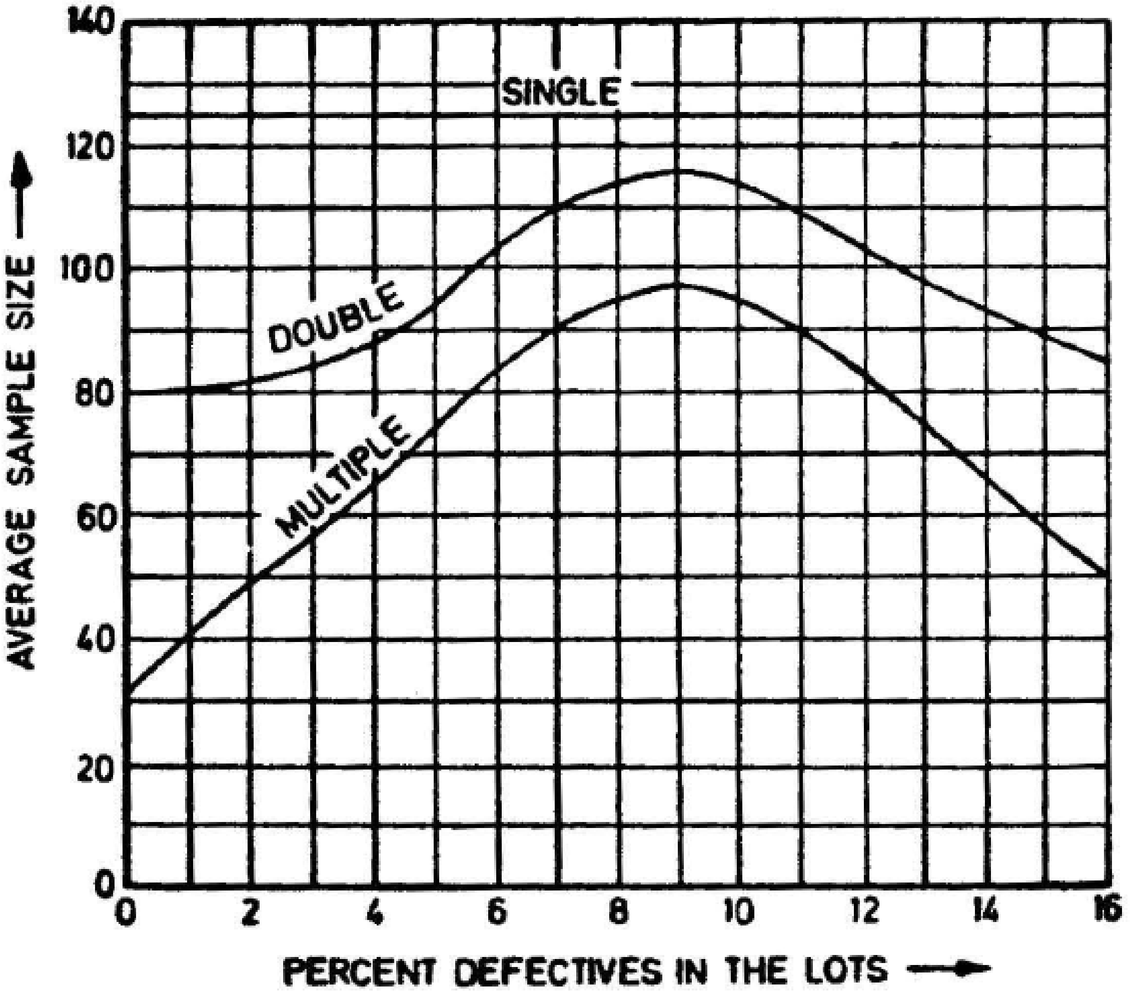


FIG. 12 TYPICAL ASN-CURVES

ANNEX A

(*Clause 2*)

LIST OF REFERRED STANDARDS

IS No.	*Title*
IS 2500 (Part 1) : 2000/ISO 2859-1 : 1999	Sampling procedures for inspection by attributes: Part 1 Sampling schemes indexed by acceptance quality limit (AQL) for lot-by-lot inspection (*third revision*)
IS/ISO 3951 (Part 1) : 2022	Sampling procedures for inspection by variables: Part 1 Specification for single sampling plans indexed by acceptance quality limit (AQL) for lot-by-lot inspection for a single quality characteristic and a single AQL (*first revision*)
IS 4905 : 2015/ ISO 24153 : 2009	Random sampling and randomization procedures (*first revision*)
IS 5002 : 2024	Determination of sample size to estimate the average quality of a lot or process — Methods
IS 7920 (Part 2) : 2012/ISO 3534-2 : 2006	Statistics — Vocabulary and symbols: Part 2 Applied statistics (*third revision*)

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ANNEX B

(*Foreword*)

COMMITTEE COMPOSITION

Statistical Methods for Quality, Data Analytics and Reliability Sectional Committee, MSD 03

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Amendments Issued Since Publication

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