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Optics and photonics — Preparation of drawings for optical elements and systems —

Part 12:
Aspheric surfaces

Optique et photonique — Préparation des dessins pour éléments et systèmes optiques —

Partie 12: Surfaces asphériques

Reference number
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Contents

Page

Foreword

ISO (the International Organization for Standardization) is a worldwide federation of national standards bodies (ISO member bodies). The work of preparing International Standards is normally carried out through ISO technical committees. Each member body interested in a subject for which a technical committee has been established has the right to be represented on that committee. International organizations, governmental and non-governmental, in liaison with ISO, also take part in the work. ISO collaborates closely with the International Electrotechnical Commission (IEC) on all matters of electrotechnical standardization.

The procedures used to develop this document and those intended for its further maintenance are described in the ISO/IEC Directives, Part 1. In particular, the different approval criteria needed for the different types of ISO documents should be noted. This document was drafted in accordance with the editorial rules of the ISO/IEC Directives, Part 2 (see www.iso.org/directives).

Attention is drawn to the possibility that some of the elements of this document may be the subject of patent rights. ISO shall not be held responsible for identifying any or all such patent rights. Details of any patent rights identified during the development of the document will be in the Introduction and/or on the ISO list of patent declarations received (see www.iso.org/patents).

Any trade name used in this document is information given for the convenience of users and does not constitute an endorsement.

For an explanation of the voluntary nature of standards, the meaning of ISO specific terms and expressions related to conformity assessment, as well as information about ISO's adherence to the World Trade Organization (WTO) principles in the Technical Barriers to Trade (TBT) see www.iso.org/iso/foreword.html.

This document was prepared by Technical Committee ISO/TC 172, *Optics and photonics*, Subcommittee SC 1, *Fundamental standards*.

This third edition cancels and replaces the second edition (ISO 10110-12:2007), which has been technically revised. It also incorporates the Amendment ISO 10110-12:2007/Amd.1:2013.

The main changes compared to the previous edition are as follows:

a) The document has been updated with respect to surface form tolerances as described in ISO 10110-5.

b) The reference to the new part ISO 10110-19 has been added.

c) The document has been restructured.

d) A few surface descriptions have been added.

A list of all the parts in the ISO 10110 series can be found on the ISO website.

Any feedback or questions on this document should be directed to the user's national standards body. A complete listing of these bodies can be found at www.iso.org/members.html.

Optics and photonics — Preparation of drawings for optical elements and systems —

Part 12:
Aspheric surfaces

1 Scope

This document specifies rules for presentation of aspheric surfaces and surfaces with low order symmetry such as cylinders and toroids in the ISO 10110 series, which standardizes drawing indications for optical elements and systems. It also specifies sign conventions and coordinate systems.

This document does not apply to off-axis aspheric and discontinuous surfaces such as Fresnel surfaces or gratings.

NOTE For off-axis aspheric and non-symmetric surfaces, see ISO 10110-19.

This document does not specify the method by which conformity with the specifications is tested.

2 Normative references

The following documents are referred to in the text in such a way that some or all of their content constitutes requirements of this document. For dated references, only the edition cited applies. For undated references, the latest edition of the referenced document (including any amendments) applies.

ISO 1101:2017, *Geometrical product specifications (GPS) — Geometrical tolerancing — Tolerances of form, orientation, location and run-out*

ISO 10110-1, *Optics and photonics — Preparation of drawings for optical elements and systems — Part 1: General*

ISO 10110-5, *Optics and photonics — Preparation of drawings for optical elements and systems — Part 5: Surface form tolerances*

ISO 10110-6, *Optics and photonics — Preparation of drawings for optical elements and systems — Part 6: Centring tolerances*

ISO 10110-7, *Optics and photonics — Preparation of drawings for optical elements and systems — Part 7: Surface imperfections*

ISO 10110-8, *Optics and photonics — Preparation of drawings for optical elements and systems — Part 8: Surface texture*

3 Terms and definitions

No terms and definitions are listed in this document.

ISO and IEC maintain terminological databases for use in standardization at the following addresses:

— ISO Online browsing platform: available at https://www.iso.org/obp

— IEC Electropedia: available at http://www.electropedia.org/

4 Mathematical description of aspheric surfaces

4.1 Coordinate system

Aspheric surfaces are described in a right-handed, orthogonal coordinate system in which the Z axis is the optical axis.

Unless otherwise specified, the Z axis is in the plane of the drawing and runs from left to right; if only one cross section is drawn, the Y axis is in the plane of the drawing and is oriented upwards.

The origin of the coordinate system is at the vertex of the aspheric surface (see Figure 1).

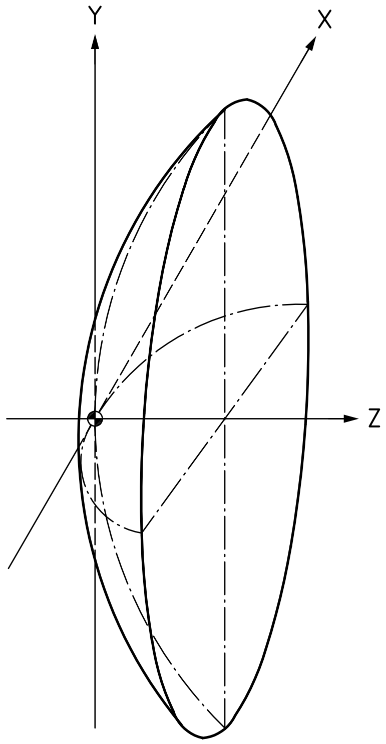


Figure 1 — Coordinate system

4.2 Sign conventions

As shown later in this document, the various types of aspheric surfaces are given by mathematical formulae. In the drawings the chosen formula and the corresponding constants and coefficients are specified. To achieve unambiguous indications of the surfaces, sign conventions for the constants and coefficients shall be introduced.

The sign of the radius of curvature is positive if the centre of curvature is to the right of the vertex, and negative if the centre of curvature is to the left of the vertex.

The sagitta of a point of the aspheric surface is positive if this point is to the right of the vertex (XY plane) and negative if it is to the left of the vertex (XY plane).

NOTE 1 In this case, “left” and “right” presume Z is increasing from left to right. When the Z axis is reversed as a result of a reflection (a 180-degree rotation about the Y axis), the sign convention for radius and sagitta is also reversed. This is discussed further in 4.3.

NOTE 2 This is the default sign convention, assuming no coordinate system according to ISO 10110-1:2019, 5.3 has been defined for the surface of interest. See ISO 10110-1 for more information about defining local coordinate systems.

4.3 Surface descriptions

4.3.1 General

The phrase “aspheric surfaces” is commonly used in optics to describe rotationally invariant surfaces such as are described below in 4.3.2. Surface descriptions for surfaces which are not rotationally invariant such as cylindrical surfaces are described in 4.3.3. More complex optical surfaces can be described using the methods given in ISO 10110-19.

4.3.2 Surface description — Rotationally invariant ($h^2 = x^2 + y^2$)

4.3.2.1 Aspheric surface described by a conic section and a power series

The aspheric surface description consists of a conic part and a power series where the axis of rotation is the Z axis.

$$z(h) = \frac{h^2}{R\left[1+\sqrt{1-(1+\kappa)\left(\frac{h}{R}\right)^2}\right]} + \sum_{i=2}^{n} A_{2i} h^{2i} \quad (1)$$

where

- z is the sagitta;
- h is surface height perpendicular to Z-axis ($h \geq 0$);
- R is the radius of curvature of the base sphere;
- κ is the conic constant;
- A_i is the aspheric coefficient.

Where the basic conic formula behaves as follow:

$\kappa > 0$	oblate ellipse;
$\kappa = 0$	circle;
$-1 < \kappa < 0$	prolate ellipse;
$\kappa = -1$	parabola;
$\kappa < -1$	hyperbola.

NOTE 1 The formula of second order can also be used without the power series.

NOTE 2 In three dimensions, the conic formula shapes are called ellipsoid, sphere, paraboloid, and hyperboloid.

For an example drawing, see Figure 4.

An extended version with the complete power series of this description is described in Formula (2).

$$z(h)=\frac{h^2}{R\left[1+\sqrt{1-(1+\kappa)\left(\frac{h}{R}\right)^2}\right]}+\sum_{i=1}^{n}A_\mathrm{i}h^\mathrm{i} \tag{2}$$

In a special version this formula describes an axicon:

$$z(h)=A_1h \tag{3}$$

Care should be taken that the signs of the power series named formulae $z(h)$ are in accordance with the conventions defined in 4.1 and 4.2.

In the case where the direction of the Z axis is reversed, but the lens stays unchanged, the signs of the radius of curvature and of the aspheric coefficients shall be changed. The signs of the conic constants remain unchanged.

4.3.2.2 Aspheric surface described by a conic section and orthogonal polynomials

4.3.2.2.1 Orthonormal in slope aspheres with spherical base

A surface of higher order can also be generated by combining a spherical surface with a polynomial of the following kind, which has orthonormal derivatives.

$$z(h)=\frac{h^2}{R\left[1+\sqrt{1-\left(\frac{h}{R}\right)^2}\right]}+\frac{w^2\left[1-w^2\right]}{\sqrt{1-\left(\frac{h}{R}\right)^2}}\sum_{m=0}^{n}B_mQ_m^\mathrm{bfs}\left(w^2\right) \tag{4}$$

where

R is the radius of curvature of the base sphere;

h is the surface height;

h_0 marks the upper limit of h; and

w (normalized surface height) is defined as $w=\frac{h}{h_0}$;

B_m is the coefficient; and

$Q_\mathrm{m}^\mathrm{bfs}$ is the polynomial term.

The description z is valid for $0 \le h \le h_0$ only. The formula for the polynomial terms is

$$Q_{m+1}^\mathrm{bfs}\left(w^2\right)=\left[P_{m+1}\left(w^2\right)-g_mQ_m^\mathrm{bfs}\left(w^2\right)-k_{m-1}Q_{m-1}^\mathrm{bfs}\left(w^2\right)\right]/\,l_{m+1} \tag{5}$$

starting with

$$Q_0^\mathrm{bfs}\left(w^2\right)=1 \tag{6}$$

$$Q_1^\mathrm{bfs}\left(w^2\right)=\frac{1}{\sqrt{19}}\left(13-16w^2\right) \tag{7}$$

$$Q_2^{\text{bfs}}\left(w^2\right)=\sqrt{\frac{2}{95}}\left[29-4w^2\left(25-19w^2\right)\right] \quad (8)$$

$$P_{m+1}\left(w^2\right)=\left(2-4w^2\right)P_m\left(w^2\right)-P_{m-1}\left(w^2\right) \quad (9)$$

starting with

$$P_0\left(w^2\right)=2 \quad (10)$$

$$P_1\left(w^2\right)=6-8w^2 \quad (11)$$

These auxiliary polynomials have to be solved in the order given here and are valid for $m \geq 2$.

$$k_{m-2}=-m(m-1)/2l_{m-2} \quad (12)$$

$$g_{m-1}=-\left(1+g_{m-2}k_{m-2}\right)/l_{m-1} \quad (13)$$

$$l_m=\left[m(m+1)+3-g_{m-1}^2-k_{m-2}^2\right]^{1/2} \quad (14)$$

starting with

$$g_0=-\frac{1}{2} \quad (15)$$

$$l_0=2 \quad (16)$$

$$l_1=\frac{1}{2}\sqrt{19} \quad (17)$$

NOTE 1 $Q_2^{\text{bfs}}\left(w^2\right)$ is given above to be used as a check of the recursion algorithm provided in Formulae (5) through (17). See also Annex B.

NOTE 2 "bfs" is an abbreviation for "best fit sphere", which matches the sag of the aspherical surface at the vertex and h_0.

For an example drawing, see Figure 5.

4.3.2.2.2 Orthonormal in slope aspheres with conic base

It is also possible to generate a surface by combining a conical surface with a polynomial of the same kind as in Formula (4). This kind is also an orthonormal set of polynomials.

$$z(h)=\frac{h^2}{R\left[1+\sqrt{1-(1+\kappa)\left(\frac{h}{R}\right)^2}\right]}+\frac{w^2\left[1-w^2\right]}{\sqrt{1-\left(\frac{h}{R}\right)^2}}\sum_{m=0}^{n}B_mQ_m^{\text{bfs}}\left(w^2\right) \quad (18)$$

where

R	is the radius of curvature of the base sphere;
h	is the surface height;
h_0	marks the upper limit of h;
w	(normalized surface height) is defined as $w = \frac{h}{h_0}$;
κ	is the conic constant;
B_m	are the coefficients; and
$Q_\mathrm{m}^\mathrm{bfs}$	are the polynomial terms.

4.3.2.2.3 Orthonormal in amplitude aspheres

A surface of higher order can also be generated by combining a conical surface with a polynomial of the following kind, which has orthonormal amplitudes.

$$z(h) = \frac{h^2}{R\left[1+\sqrt{1-(1+\kappa)\left(\frac{h}{R}\right)^2}\right]} + w^4 \sum_{m=0}^{n} C_m Q_m^\mathrm{con}\left(w^2\right) \tag{19}$$

where

R	is the radius of curvature of the base sphere;
h	is the surface height;
h_0	marks the upper limit of h; and
w	(normalized surface height) is defined as $w = \frac{h}{h_0}$;
κ	is the conic constant;
C_m	are the coefficients; and
$Q_\mathrm{m}^\mathrm{con}$	are the polynomial terms.

The formula for the polynomial terms is

$$Q_m^\mathrm{con}\left(w^2\right) = T_m\left(2w^2 - 1\right) \tag{20}$$

starting with

$$Q_0^\mathrm{con}\left(w^2\right) = 1 \tag{21}$$

$$Q_1^\mathrm{con}\left(w^2\right) = -\left(5 - 6w^2\right) \tag{22}$$

$$Q_2^\mathrm{con}\left(w^2\right) = 15 - 14w^2\left(3 - 2w^2\right) \tag{23}$$

$$T_m\left(w^2\right)=\left[\left(b(m)+c(m)w^2\right)T_{m-1}\left(w^2\right)-d(m)T_{m-2}\left(w^2\right)\right]/a(m) \tag{24}$$

starting with

$$T_0\left(w^2\right)=1 \tag{25}$$

$$T_1\left(w^2\right)=3\mathrm{w}^2-2 \tag{26}$$

These auxiliary polynomials have to be solved in the order given here and are valid for $m \geq 2$.

$$a(m)=2m(m+4)(2m+2) \tag{27}$$

$$b(m)=-32m-48 \tag{28}$$

$$c(m)=(2m+2)(2m+3)(2m+4) \tag{29}$$

$$d(m)=2(m-1)(m+3)(2m+4) \tag{30}$$

NOTE 1 $Q_0^{\mathrm{con}}\left(w^2\right)$, $Q_1^{\mathrm{con}}\left(w^2\right)$ and $Q_2^{\mathrm{con}}\left(w^2\right)$ are given above to be used as a check of the recursion algorithm provided in Formulae (20) through (30). See also Annex C.

NOTE 2 The polynomial form given here is identical to Zernike radial polynomial expansion $R_{2n}^{q}\left(w^2\right)$ of order q = 4 correlated to ISO/TR 14999-2 as $w^4 Q_m^{con}\left(w^2\right)=R_{2n}^{4}\left(w^2\right)$ with $n = m + 2$, and m = 0, 1, 2, 3, 4, 5, 6 ...

NOTE 3 Instead of using the recursion algorithm above, for lower orders of m ($m \leq 8$), the polynomials can easily be computed using the formula $Q_m^{con}\left(w^2\right)=\sum_{s=0}^{m}(-1)^s\frac{(2m+4-s)!}{s!(m+4-s)!(m-s)!}w^{2(m-s)}$.

For an example drawing, see Figure 6.

4.3.3 Surface description — Rotationally variant

4.3.3.1 Centred quadrics

In the coordinate system given in 4.1, the formulae of the surfaces of second order which fall within the scope of this document are derived from the canonical forms

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1 \quad \text{for centered quadrics} \tag{31}$$

where

a, b are real or imaginary constants;

c is a real constant.

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}+2z=0 \qquad \text{for parabolic surfaces} \tag{32}$$

where a, b are real or imaginary constants, and can be written as

$$z=f(x,y)=\frac{\dfrac{x^2}{R_X}+\dfrac{y^2}{R_Y}}{1+\sqrt{1-(1+\kappa_X)\left(\dfrac{x}{R_X}\right)^2-(1+\kappa_Y)\left(\dfrac{y}{R_Y}\right)^2}} \tag{33}$$

where

R_X is the radius of curvature in the XZ plane;

R_Y is the radius of curvature in the YZ plane;

κ_X, κ_Y are conic constants.

Using curvatures $C_X = 1/R_X$ and $C_Y = 1/R_Y$ instead of radii yields

$$z=f(x,y)=\frac{x^2C_X+y^2C_Y}{1+\sqrt{1-(1+\kappa_X)(xC_X)^2-(1+\kappa_Y)(yC_Y)^2}} \tag{34}$$

If the surface according to Formula (33) or (34) is intersected with the XZ plane (y = 0) or the YZ plane (x = 0), then, depending on the value of κ_Y (or κ_X), intersection lines of the following types are produced:

$\kappa > 0$ oblate ellipse;

$\kappa = 0$ circle;

$-1 < \kappa < 0$ prolate ellipse;

$\kappa = -1$ parabola;

$\kappa < -1$ hyperbola.

4.3.3.2 Cylinders

Formulae (35) and (36) describe a cylinder (due to κ_U not necessarily of circular cross section). For $u = x$, the cylinder vertex line is parallel to the Y axis which is perpendicular to the XZ plane. For $u = y$ the cylinder vertex line is parallel to the X axis which is perpendicular to the YZ plane.

Using radii:

For $R_X = \infty$ or $R_Y = \infty$ Formula (33) gives

$$z=f(u)=\frac{u^2}{R_U\left[1+\sqrt{1-(1+\kappa_U)\left(\dfrac{u}{R_U}\right)^2}\right]} \tag{35}$$

Using curvatures:

For $C_X = 0$ or $C_Y = 0$ Formula (34) gives

$$z = f(u) = \frac{u^2 C_U}{1 + \sqrt{1 - (1 + \kappa_U) u^2 C_U^2}} \tag{36}$$

4.3.3.3 Polynomials

The formula for polynomial surfaces is

$$z = f(x, y) = A_2 x^2 + B_2 y^2 + A_4 x^4 + B_4 y^4 + A_6 x^6 + B_6 y^6 + \ldots + C_3 |x|^3 + \ldots + D_3 |y|^3 + \ldots \tag{37}$$

Alternatively, this formula can be shown as a summation

$$z = f(x, y) = \sum_{i=2}^{n} \left(\mathrm{A}_i |x|^i + B_i |y|^i \right) \tag{38}$$

4.3.3.4 Toric surfaces

A toric surface is generated by the rotation of a defining curve, contained in a plane, about an axis which lies in the same plane.

The formula of a toric surface having its defining curve, $z = g(x)$, in the XZ plane and its axis of rotation parallel to the X axis is

$$z = f(x, y) = R_\mathrm{Y} \mp \sqrt{[R_\mathrm{Y} - g(x)]^2 - y^2} \tag{39}$$

where R_Y is the z-coordinate at which the axis of rotation intersects the Z axis.

For the purpose of this document, $g(x)$ is derived from Formula (33) by setting $y = 0$.

$$g(x) = \frac{x^2}{R_X \left[1 + \sqrt{1 - (1 + \kappa_X) \left(\frac{x}{R_X} \right)^2} \right]} \tag{40}$$

The formula of a toric surface having its defining curve in the YZ plane and its axis of rotation parallel to the Y axis may be obtained from Formulae (39) and (40) by interchanging x with y, R_X with R_Y and κ_X with κ_Y.

The following special case of Formulae (39) and (40) should be mentioned:

If $\kappa_X = 0$

$$g(x) = R_X \left[1 - \sqrt{1 - \left(\frac{x}{R_X} \right)^2} \right] \tag{41}$$

and

$$f(x, y) = R_Y \mp \sqrt{\left[R_Y - R_X + R_X \sqrt{1 - \left(\frac{x}{R_X} \right)^2} \right]^2 - y^2} \tag{42}$$

Formula (42) describes a torus whose defining curve is a circle with radius R_X.

4.3.3.5 Conical surfaces

The canonical form

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=0 \tag{43}$$

where

a, b are imaginary constants;

c is a real constant;

leads to Formula (44)

$$z=f(x,y)=c\sqrt{\frac{x^2}{a^2}+\frac{y^2}{b^2}} \tag{44}$$

where a, b, c are real constants.

This formula describes a cone with its tip at the origin, with elliptical cross section (if $a \neq b$) or with circular cross section (if $a = b$).

4.3.3.6 Combinations of surface types

If necessary, surface types can be modified by the addition of another function $f_1(x, y)$. The complete formula of the surface is then

$$z=f(x,y)+f_1(x,y) \tag{45}$$

where

$f(x,y)$ where represents the basic form according to Formulae (33) and (34) or one of their derivatives and

$f_1(x,y)$ represents polynomial surface functions like Formula (37).

For cylindrical surfaces, the defining curve $f(u)$ can be modified by addition of a power series $f_1(u)$ (see Annex A.) For toric surfaces, the defining curve $g(x)$ can be modified by addition of a power series $g_1(x)$ (see Annex A.).

Care should be taken that the signs of the coefficients [Formula (45)] in $f(x, y)$ and $f_1(x, y)$ [or for toric surfaces the signs of the coefficients $g(x, y)$ and $g_1(x, y)$] are in accordance with the conventions defined in 4.1 and 4.2. In the case where the direction of the Z axis shall be reversed, the signs of the radii and curvatures and of the coefficients for Formula (37) or (38) shall be changed. The signs of the conic constants remain unchanged.

5 Indications in drawings

5.1 Indication of the theoretical surface

The formula describing the aspheric surface shall be given.

The radius of curvature is indicated with a sign, in accordance with 4.2.

The surface type shall be given as described in ISO 10110-1.

A sagitta table having sufficient numerical accuracy shall be included on the drawing (see Figures 2 and 3).

5.2 Indication of surface form tolerances

Surface form tolerances shall be indicated in one of the following ways:

a) in accordance with ISO 1101; or

b) in accordance with ISO 10110-5;

NOTE The specification in version ISO 10110-12:2007 has been adapted and moved to ISO 10110-5.

5.3 Indication of centring tolerances

Centring tolerances shall be indicated in accordance with either ISO 1101 or ISO 10110-6.

5.4 Indication of surface imperfection and surface texture tolerances

Tolerances for surface imperfections shall be indicated according to ISO 10110-7. Specifications of mid-spatial frequency ripple and surface texture shall be indicated according to ISO 10110-8.

6 Examples

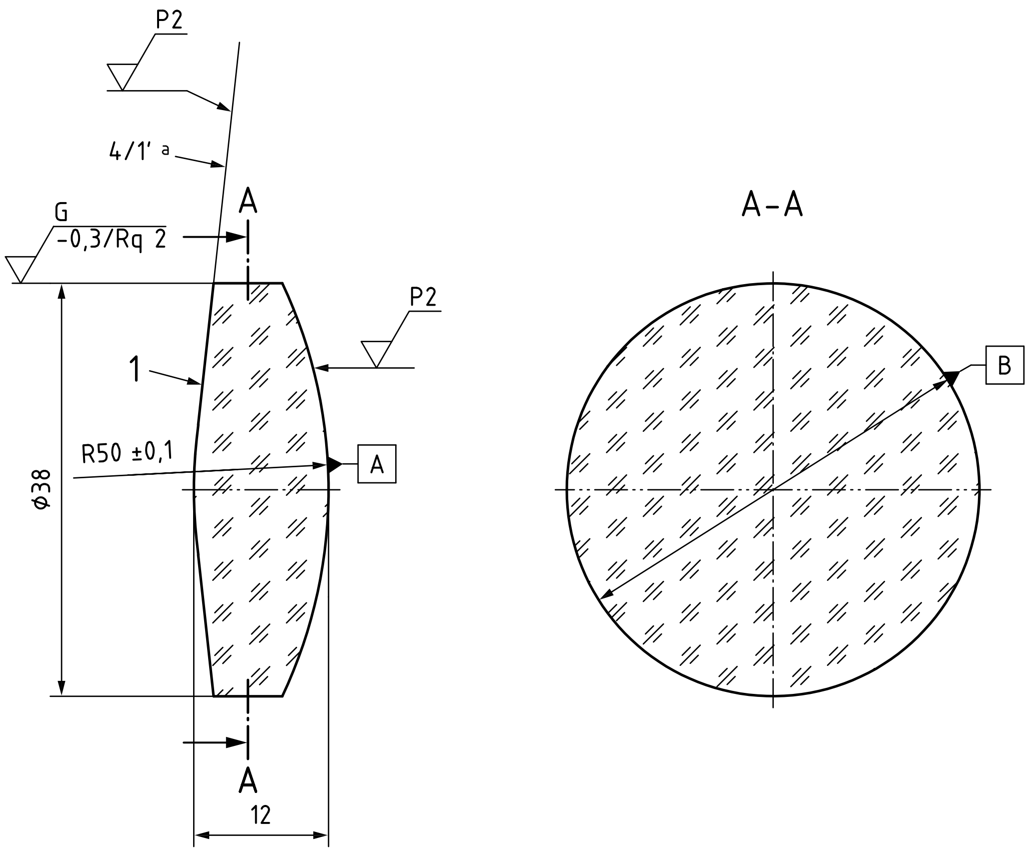
6.1 Parts with rotationally invariant surfaces

In Figure 2, an aspheric lens is shown, where the datum axis is established by centre of curvature of spherical surface A and the outer cylinder axis B determined at the circular target line, which is located at the intersection of the spherical surface with the outer cylinder in accordance with ISO 10110-6.

The form tolerance of the aspheric surface is given in tabular form. Δz is the maximum permissible deviation, in millimetres, in the Z direction for the given h coordinate. In addition, a slope deviation ΔS tolerance is indicated.

The centring tolerance is indicated in accordance with ISO 1101 as the maximum permissible axial run-out, and, alternatively, in accordance with ISO 10110-6 as the maximum permissible tilt angle (marked with index a).

Dimensions in millimetres



Key

1 asphere

$$z = \frac{h^2}{R\left(1+\sqrt{1-(1+\kappa)h^2/R^2}\right)} + \sum_{i=2}^{5}\left(A_{2i}h^{2i}\right)$$

a Alternative indication of centring tolerance.

h	Z	Δz	ΔS
0,0	0,000	0,000	0,3'
5,0	0,219352	0,002	0,5'
10,0	0,825330	0,004	0,5'
15,0	1,600528	0,006	0,8'
19,0	1,938077	0,008	
			(1/0,1)

R = 56,031

κ = −3

A_4 = −0,43264 E-05

A_6 = −0,97614 E-08

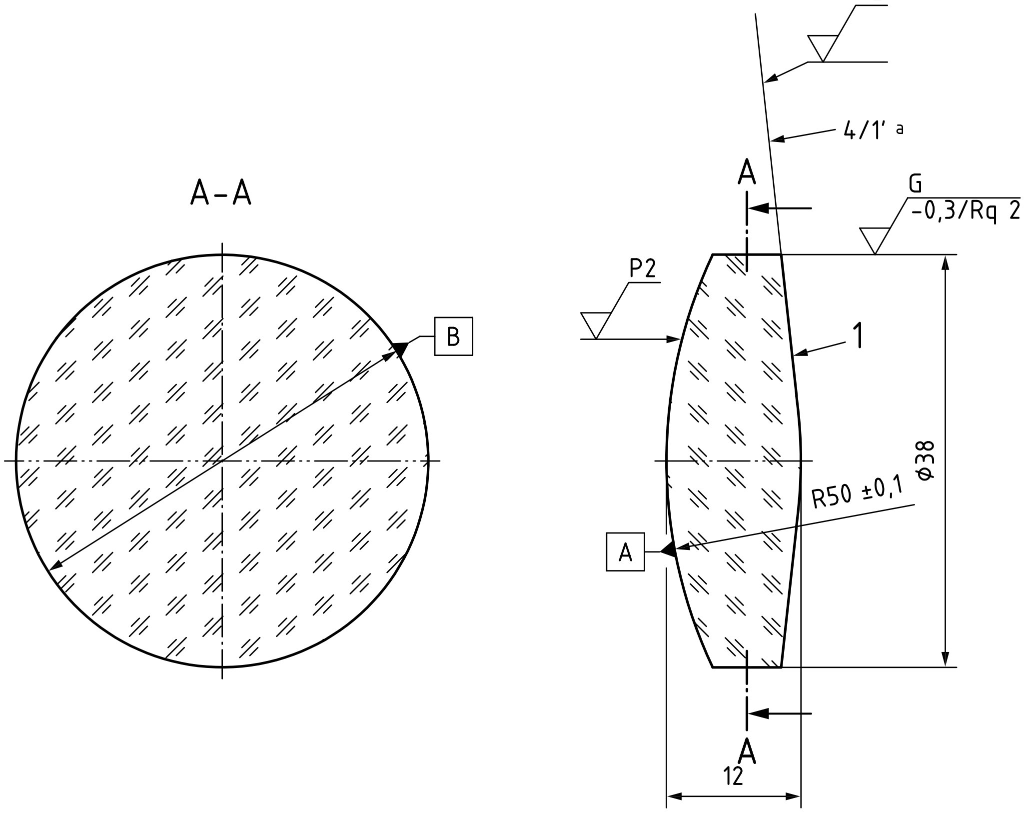
A_8 = −0,10852 E-11

A_{10} = −0,12284 E-13

Figure 2 — Lens with a rotationally invariant aspheric surface

In Figure 3 the drawing of an aspherical lens with the same geometrical shape as the lens in Figure 2 but turned over (so that the asphere is the right surface) is shown. Note that the signs of the radius, R and of the coefficients A_i have changed. As a result the sagittas, z, have also changed sign.

Dimensions in millimetres



Key

1 asphere

$$z=\frac{h^2}{R\left(1+\sqrt{1-(1+\kappa)h^2/R^2}\right)}+\sum_{i=2}^{5}\left(A_{2i}h^{2i}\right)$$

[a] Alternative indication of centring tolerance.

h	Z	Δz	ΔS
0,0	0,000	0,000	0,3'
5,0	−0,219352	0,002	0,5'
10,0	−0,825330	0,004	0,5'
15,0	−1,600528	0,006	0,8'
19,0	−1,938077	0,008	
			(1/0,1)

R = −56,031

κ = −3

A_4 = 0,43264 E-05

A_6 = 0,97614 E-08

A_8 = 0,10852 E-11

A_{10} = 0,12284 E-13

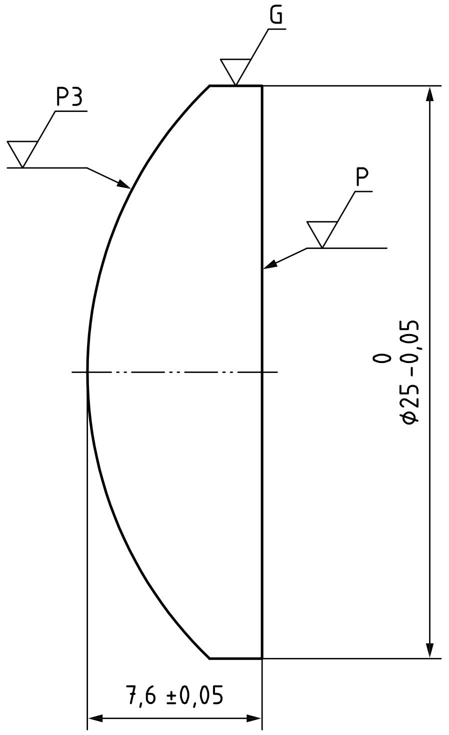
Figure 3 — Lens with a reversed rotationally invariant aspheric surface

$$z_3(h) = \frac{h^2}{R\left[1+\sqrt{1-(1+\kappa)\left(\frac{h}{R}\right)^2}\right]} + \sum_{i=2}^{n} A_{2i} h^{2i}$$

R = 15,54 ±0,5%
κ = -1,35
A_4 = 2,3618134e-005
A_6 = -1,1303079e-008
A_8 = -1,1113906e-011
A_{10} = -2,3981714e-014
A_{12} = 3,035791e-017
A_{14} = 1,3660815e-019
A_{16} = -1,8881587e-022

h	$z(h)$
0,0	0,000000
2,0	0,128891
4,0	0,517849
6,0	1,173636
8,0	2,107112
10,0	3,332245
12,0	4,863958
12,5	5,296338

1) all dimensions in mm



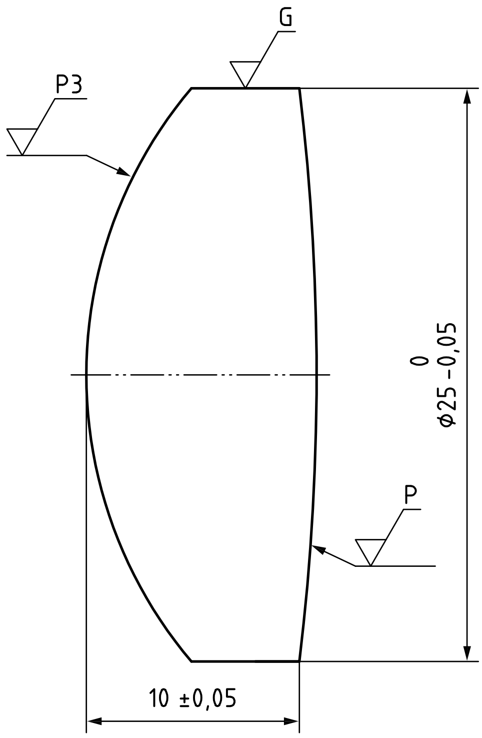
Surface 1	Material	Surface 2
ASPH	Company / S-LAH64	$R = \infty$
ϕ_e 22,5	n_d = 1,78800	ϕ_e 23
3/-(10)RMSi < 0,5 µm;	ν_d = 47,36853	protective chamfer 0,2-0,4
$\Delta S_{1\text{-dim},x}$ (100 µrad/1/0,1)	0/-	3/5 (0,5)
4/5′	1/2 × 0,1	4/-
5/2 × 0,25; L1 × 0,1; E0,3	2/-	5/2 × 0,25; L1 × 0,1; E0,3
		A4 / Scale 3:1 Ind. Acc. ISO 10110, λ = 632 nm

Figure 4 — Description of an aspherical surface with conic and power series

$$z(h)=\frac{h^2}{R\left[1+\sqrt{1-\left(\frac{h}{R}\right)^2}\right]}+\frac{\left(\frac{h}{h_0}\right)^2\left[1-\left(\frac{h}{h_0}\right)^2\right]}{\sqrt{1-\left(\frac{h}{R}\right)^2}}\sum_{m=0}^{n}B_m Q_m^{\text{bfs}}\left(\frac{h^2}{h_0^2}\right)$$

R = 20 ±0,5%
h_0 = 14
B_0 = 0,677
B_1 = −0,029

h	$z(h)$
0,0	0,000000
2,0	0,112160
4,0	0,449930
6,0	1,017104
8,0	1,819724
10,0	2,865558
12,0	4,162932
12,5	4,527382



1) all dimensions in mm

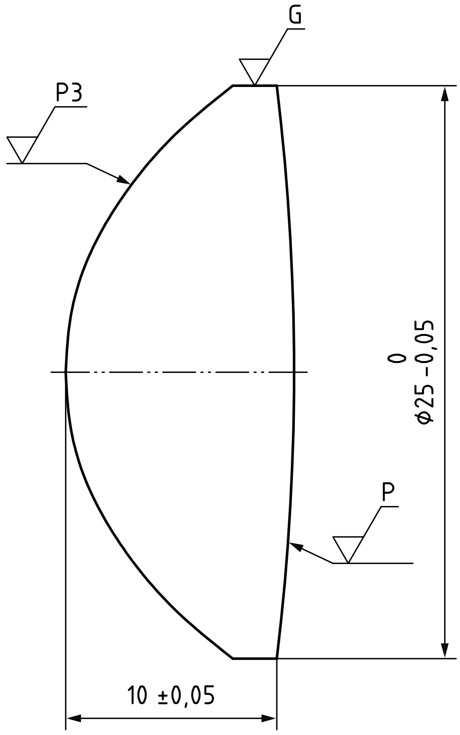
Surface 1	Material	Surface 2
ASPH	Company / N-BK7	R = 127 ± 0,5% cx
ϕ_e 22,5	n_d = 1,5168	ϕ_e 22,5
3/-(10)RMSi < 50 µm;	ν_d = 64,16734	protective chamfer 0,2-0,4
RMS$\Delta S_{2\text{-dim}}$ (50 µrad/1/0,1)	0/10	3/- (0,5)
4/5′	1/2 × 0,16	4/-
5/2 × 0,25; L1 × 0,1; E0,3	2/-	5/2 × 0,25; L1 × 0,1; E0,3
		A4 / Scale 3:1 Ind. Acc. ISO 10110, λ = 632 nm

Figure 5 — Description of an aspherical surface using an orthonormal in slope asphere description

$$z(h) = \frac{h^2}{R\left[1 + \sqrt{1 - \left(\frac{h}{R}\right)^2}\right]} + \left(\frac{h}{h_0}\right)^4 \sum_{m=0}^{n} C_m Q_m^{\mathrm{con}} \left(\frac{h^2}{h_0^2}\right)$$

R = 11,86
κ = -1,1
h_0 = 14
C_0 = 1,256
C_1 = -0,046
C_2 = -0,0107

h	$z(h)$
0,0	0,000000
2,0	0,169068
4,0	0,681543
6,0	1,553568
8,0	2,812091
10,0	4,491239
12,0	6,619876
12,5	7,223065



1) all dimensions in mm

Surface 1	Material	Surface 2
ASPH	Company / N-BK7	R = 100 ± 0,5% cx
ϕ_e 22,5	n_d = 1,5168	ϕ_e 22,5
3/-(10)RMSi < 0,5 µm;	ν_d = 64,16734	protective chamfer 0,2-0,4
$\Delta S_{\text{1-dim,x}}$ (100 µrad/1/0,1)	0/10	3/- (0,5)
4/5′	1/2 × 0,16	4/-
5/2 × 0,25; L1 × 0,1; E0,3	2/-	5/2 × 0,25; L1 × 0,1; E0,3
		A4 / Scale 3:1 Ind. Acc. ISO 10110, λ = 632 nm

Figure 6 — Description of an aspherical surface using an orthonormal in amplitude asphere description

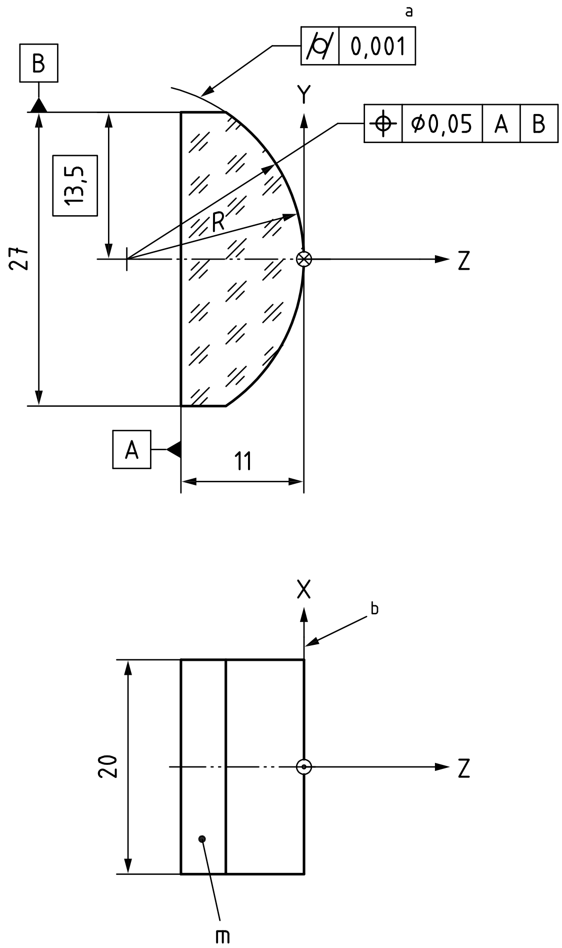
6.2 Parts with rotationally variant surfaces

Figure 7 shows a planocylinder lens with rectangular cross section. The datum axis is given by the intersection of surfaces A and B.

The axis of the cylindrical surface shall be within a cylinder of diameter 0,05 mm.

The form error tolerance is specified in accordance with ISO 1101:2017, 17.5 and additionally by different slope deviation tolerances in the two sections.

Dimensions in millimetres



Key

m mark for identification

R = −17,2 ± 0,2

a $3/\Delta S_{\text{1-dim,Y}}(0{,}5'/2/0{,}2)$

b $3/\Delta S_{\text{1-dim, X}}\,(1{,}0'/2/0{,}2)$

Figure 7 — Planocylinder lens

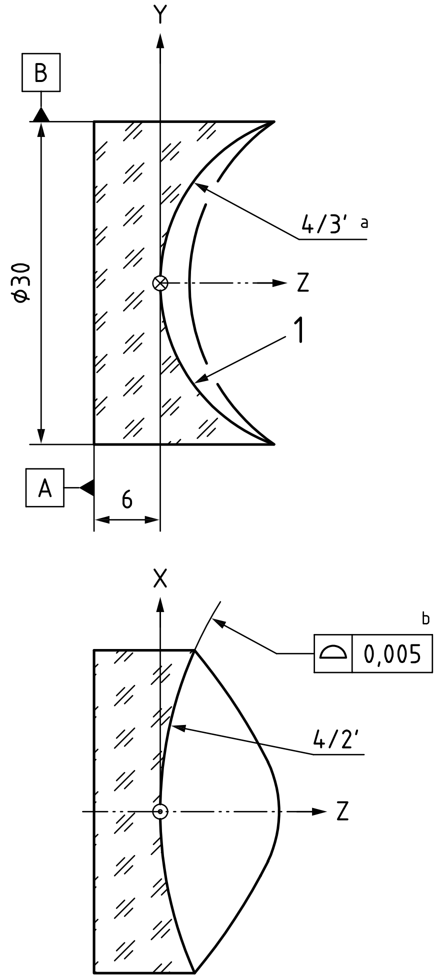
Figure 8 shows a planotoric lens with circular cross section.

The datum axis is given by the edge cylinder B and the plano surface A.

The surface formula shown in the drawing indicates that defining arc and rotation axis of the surface lie in the XZ plane.

Different tolerances for the surface tilt angles are given in the two cross sections. Also, the (local) slope angle tolerances are different in the two cross sections.

Dimensions in millimetres



Key

1 TORIC

$$z = R_Y - \sqrt{\left[R_Y - R_X + \sqrt{R_X^2 - x^2} \right]^2 - y^2}$$

R_Y = 16 ± 0,1

R_X = 40 ± 0,2

a 3/$\Delta S_{\text{1-dim, Y}}$ (0,5′/3/0,2)

b 3/$\Delta S_{\text{1-dim, X}}$ (0,8′/3/0,2)

Figure 8 — Planotoric lens

Annex A
(informative)

Summary of aspheric surface types

See Table A.1.

Table A.1

Class	Basic surface type	Basic surface function	Additional power series term/polynomial
Rotationally invariant surface about Z axis	Sphere Ellipsoid Hyperboloid Paraboloid	$\frac{h^2}{R\left[1+\sqrt{1-(1+\kappa)\left(\frac{h}{R}\right)^2}\right]}$	$\sum_{i=1}^{n} A_i h^i$ where $h=\sqrt{x^2+y^2}$
	Plane	0	
	Conoid (a = b)	$\frac{c}{a}h$	
Orthonormal in slope asphere Rotationally invariant surface about Z axis	Sphere	$\frac{h^2}{R\left[1+\sqrt{1-\left(\frac{h}{R}\right)^2}\right]}$	$\frac{w^2\left(1-w^2\right)}{\sqrt{1-\left(\frac{h}{R}\right)^2}}\sum_{i=0}^{n} B_i Q_i^{\text{bfs}}\left(w^2\right)$ where $w=\frac{h}{h_0}, h=\sqrt{x^2+y^2}$ Definition of $Q_i^{\text{bfs}}\left(w^2\right)$ in 4.3.2.2.1
	Ellipsoid Hyperboloid Paraboloid	$\frac{h^2}{R\left[1+\sqrt{1-(1+\kappa)\left(\frac{h}{R}\right)^2}\right]}$	
Orthonormal in amplitude asphere Rotationally invariant surface about Z axis	Sphere Ellipsoid Hyperboloid Paraboloid	$\frac{h^2}{R\left[1+\sqrt{1-(1+\kappa)\left(\frac{h}{R}\right)^2}\right]}$	$\mathrm{w}^4\sum_{\mathrm{i}=0}^{\mathrm{n}} \mathrm{C}_\mathrm{i} \mathrm{Q}_\mathrm{i}^{\mathrm{con}}\left(\mathrm{w}^2\right)$ where $w=\frac{h}{h_0}, h=\sqrt{x^2+y^2}$ Definition of $Q_i^{\text{con}}\left(w^2\right)$ in 4.3.2.2.3

Table A.1 *(continued)*

Class	Basic surface type	Basic surface function	Additional power series term/polynomial
Rotationally variant surface	Toroid	$R_Y \mp \sqrt{(R_Y - g(x))^2 - y^2}$ with $g(x) = \dfrac{x^2}{R_X\left[1+\sqrt{1-(1+\kappa_X)\left(\dfrac{x}{R_X}\right)^2}\right]}$	$g_1(x) = \sum_{i=2}^{n} A_i \lvert x \rvert^i$
	Conoid	$\dfrac{\dfrac{x^2}{R_X}+\dfrac{y^2}{R_Y}}{1+\sqrt{1-(1+\kappa_X)\left(\dfrac{x}{R_X}\right)^2-(1+\kappa_Y)\left(\dfrac{y}{R_Y}\right)^2}}$ or $c\sqrt{\dfrac{x^2}{a^2}+\dfrac{y^2}{b^2}}$	$\sum_{i=2}^{n}\left(A_i \lvert x \rvert^i + B_i \lvert y \rvert^i\right)$
Translationally invariant surface about X or Y axis	Cylinder	$\dfrac{u^2}{R_U\left[1+\sqrt{1-(1+\kappa_U)\left(\dfrac{u}{R_U}\right)^2}\right]}$ where $u = x$ or y, $U = X$ or Y	$\sum_{i=2}^{n} A_i \lvert u \rvert^i$ where $u = x$ or y

Annex B
(informative)

Description of orthonormal in slope aspheres

The following formula describe the generation of the Q^{bfs} for 4.3.2.2.1 and 4.3.2.2.2:

$$Q_{m+1}^{\mathrm{bfs}}\left(w^2\right)=\left[P_{m+1}\left(w^2\right)-g_m Q_m^{\mathrm{bfs}}\left(w^2\right)-k_{m-1}Q_{m-1}^{\mathrm{bfs}}\left(w^2\right)\right]/\,l_{m+1} \quad \text{(B.1)}$$

$$P_{m+1}\left(w^2\right)=\left(2-4w^2\right)P_m\left(w^2\right)-P_{m-1}\left(w^2\right) \quad \text{(B.2)}$$

starting with

$$P_0\left(w^2\right)=2$$

$$P_1\left(w^2\right)=6-8w^2.$$

The following auxiliary polynomials (B.3), (B.4) and (B.5) have to be solved in the order given here and are valid for $m \geq 2$.

$$k_{m-2}=-m\left(m-1\right)/\,2l_{m-2} \quad \text{(B.3)}$$

$$g_{m-1}=-\left(1+g_{m-2}k_{m-2}\right)/\,l_{m-1} \quad \text{(B.4)}$$

$$l_m=\left[m\left(m+1\right)+3-g_{m-1}^2-k_{m-2}^2\right]^{1/2} \quad \text{(B.5)}$$

starting with

$$g_0=-\frac{1}{2} \quad \text{(B.6)}$$

$$l_0=2 \quad \text{(B.7)}$$

$$l_1=\frac{1}{2}\sqrt{19} \quad \text{(B.8)}$$

Based on the recursion the first six Qs are the following:

$$Q_0^{\mathrm{bfs}}\left(w^2\right)=1$$

$$Q_1^{\mathrm{bfs}}\left(w^2\right)=\frac{1}{\sqrt{19}}\left(13-16w^2\right)$$

$$Q_2^{\mathrm{bfs}}\left(w^2\right)=\sqrt{\frac{2}{95}}\left[29-4w^2\left(25-19w^2\right)\right]$$

$$Q_3^{\text{bfs}}\left(w^2\right)=\sqrt{\frac{2}{2\,545}}\left\{207-4w^2\left[315-w^2\left(577-320w^2\right)\right]\right\}$$

$$Q_4^{\text{bfs}}\left(w^2\right)=\frac{1}{3\sqrt{131\,831}}\left(7\,737-16w^2\left\{4\,653-2w^2\left[7\,381-8w^2\left(1\,168-509w^2\right)\right]\right\}\right)$$

$$Q_5^{\text{bfs}}\left(w^2\right)=\frac{1}{3\sqrt{6\,632\,213}}\left[66\,657-32w^2\left(28\,338-w^2\left\{135\,325-8w^2\left[35\,884-w^2\left(34\,661-12\,432w^2\right)\right]\right\}\right)\right]$$

Annex C
(informative)

Description of orthonormal in amplitude aspheres

The following formulae describe the generation of the Q^{con} for 4.3.2.2.3:

$$Q_m^{con}\left(w^2\right)=T_m\left(2w^2-1\right) \tag{C.1}$$

starting with

$$Q_0^{con}\left(w^2\right)=1 \tag{C.2}$$

$$Q_1^{con}\left(w^2\right)=-\left(5-6w^2\right) \tag{C.3}$$

$$Q_2^{con}\left(w^2\right)=15-14w^2\left(3-2w^2\right) \tag{C.4}$$

$$T_m\left(w^2\right)=\left[\left(b(m)+c(m)w^2\right)T_{m-1}\left(w^2\right)-d(m)T_{m-2}\left(w^2\right)\right]/a(m) \tag{C.5}$$

starting with

$$T_0\left(w^2\right)=1 \tag{C.6}$$

$$T_1\left(w^2\right)=3\mathrm{w}^2-2 \tag{C.7}$$

These auxiliary polynomials have to be solved in the order given here and are valid for $m \geq 2$.

$$a(m)=2m(m+4)(2m+2) \tag{C.8}$$

$$b(m)=-32m-48 \tag{C.9}$$

$$c(m)=(2m+2)(2m+3)(2m+4) \tag{C.10}$$

$$d(m)=2(m-1)(m+3)(2m+4) \tag{C.11}$$

Based on the recursion the first six Qs are the following:

$$Q_0^{con}\left(w^2\right)=1$$

$$Q_1^{con}\left(w^2\right)=-\left(5-6w^2\right)$$

$$Q_2^{con}\left(w^2\right)=15-14w^2\left(3-2w^2\right)$$

$$Q_3^{con}\left(w^2\right)=-\left(35-12w^2\left(14-w^2\left(21-10w^2\right)\right)\right)$$

$$Q_4^{\text{con}}\left(w^2\right)=70-3w^2\left(168-5w^2\left(84-11w^2\left(8-3w^2\right)\right)\right)$$

$$Q_5^{\text{con}}\left(w^2\right)=-\left(126-w^2\left(1\,260-11w^2\left(420-w^2\left(720-13w^2\left(45-14w^2\right)\right)\right)\right)\right)$$

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